Particle creation in strong scalar fields

or Secularly growing loop corrections in the Yukawa model

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Motivation

- Common wisdom is that loop corrections do not bring anything substantially new to the Schwinger pair creation process
- It was shown a few years ago that this is not the case[1] for massive charged scalars on the strong electric field background
- One has to solve the Dyson-Schwinger equation to completely understand this process
- This has not been done so far in QED due to the technical problems
- We consider a simplified model of QED, a **Yukawa model** on the strong **scalar field background**
- We are going to reproduce the results of the papers[1], clarify the details and further solve the DS equation

[1] 1405.5285, 1412.1554

Introduction

- We consider the Yukawa model in $(1+1)$-dim Minkowski space-time:

$$S = \int d^2x \left[\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \bar{\psi} \left(i\partial\!\!\!/ - m \right) \psi - \lambda \phi \bar{\psi} \psi \right]$$

- The corresponding equations of motion:

$$\begin{cases} \partial^2 \phi + \lambda \bar{\psi} \psi = 0 \\ \left(i\partial\!\!\!/ - m - \lambda \phi \right) \psi = 0, \end{cases}$$

  have a classical solution $\psi_{cl} = 0$, $\phi_{cl} = Et$

- In what follows we consider this solution as an external background for the Dirac field, i.e. we split the classical and quantum parts of the fields and calculate correlation functions of the quantum parts
- The Hamiltonian of such a theory strongly depends on time, so one expects the particle creation

Mode expansion

- We decompose the field in positive- and negative-frequency modes, which solve the following equation of motion:

$$(i\partial\!\!\!/ - m - \lambda E t)\psi^{(\pm)}(t,x) = 0,$$

obey the equal-time anti-commutation relations and tend to the **plane waves** at the past and future infinities

- We emphasize that we use the exact modes instead of the plane waves
- The positive-frequency modes look as follows:

$$\psi^{(+)}(t,x) = e^{-\frac{\pi p^2}{4\alpha}} \begin{pmatrix} D_\nu\left(z(t)\right) \\ \frac{1+i}{\sqrt{2}}\frac{p}{\sqrt{2\alpha}} D_{\nu-1}\left(z(t)\right) \end{pmatrix} e^{ipx},$$

where $\nu = -\frac{ip^2}{2\lambda E}$, $z(t) = \frac{1+i}{\sqrt{\lambda E}}(m + \lambda E t)$

- The negative-frequency modes are obtained by the **charge conjugation** of this expression

Mode expansion

- Note that technically parabolic cylinder functions **do not tend** to the plane waves at the infinity:

$$D_\nu(z) \sim \exp\left(-\frac{i}{2}\alpha t^2 - \frac{ip^2}{2\alpha}\log t\right),$$

  although they behave as plane waves at large momenta ($p \gg \sqrt{\lambda E}$):

$$D_\nu(z) \sim \frac{1}{\sqrt{2}}\left(1 + \frac{m + \lambda E t}{2|p|}\right) e^{-i|p|t}$$

- This allows one to define the positive- and negative-frequency modes, but also it indicates that the vacuum of the theory is **significantly perturbed**

Tree-level current

- Tree-level current in the **free theory** has the following form:

$$\langle \bar{\psi}\psi \rangle_{|\lambda \to 0} = -\int_{-\Lambda}^{\Lambda} \frac{dp}{2\pi} \frac{m}{\omega} = -\frac{m}{\pi} \log \frac{\Lambda}{m}$$

- In the interacting theory the mass parameter m is replaced by $M(t) = m + \lambda E t$
- Hence, one expects that the normal-ordered current in the interacting theory behaves as follows:

$$\langle : \bar{\psi}\psi : \rangle(t) \sim \int \frac{dp}{2\pi} \frac{m - M(t)}{\omega} \sim -\frac{\lambda E t}{\pi} \log \Lambda$$

- The straightforward calculations confirm this conjecture:

$$\langle : \bar{\psi}\psi : \rangle(t) \simeq \frac{\lambda E t}{\pi} \log \frac{\lambda E t}{\Lambda}$$

- Non-zero current indicates the production of charged particles

Effective action

- One obtains the effective action by integrating out the fermionic degrees of freedom in the corresponding functional integral:

$$e^{iS_{\text{eff}}} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi e^{i\int d^2x\left[\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + \bar{\psi}(i\not{\partial} - m)\psi - \lambda\phi\bar{\psi}\psi\right]}$$

- I.e. by summing up the fermionic loops with an even number of external bosonic legs (Fig. 1)

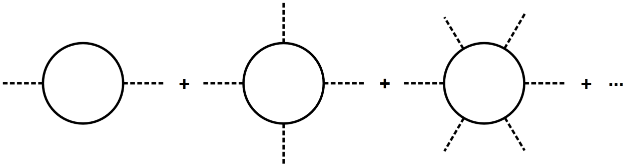

Figure: Solid lines correspond to fermions, dashed ones correspond to bosons

Effective action

- In the leading order one obtains the following effective action:

$$S_{eff} = \int d^2x \Big[\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + \bar{\psi}(i\not{\partial} - m)\psi + \frac{(\lambda\phi)^2}{2\pi}\log\frac{\Lambda}{\lambda\phi} + \frac{(\lambda\phi)^2}{4\pi} \Big]$$

- It gives the current which is consistent with calculations based on the mode expansion:

$$\langle\bar{\psi}\psi\rangle = \frac{i}{\lambda}\frac{\delta}{\delta\phi}e^{iS_{eff}}\Big|_{\phi_{cl}} = \frac{\lambda\phi_{cl}}{\pi}\log\frac{\lambda\phi_{cl}}{\Lambda}$$

- This means that particle production is a **general property** of Yukawa-like models on strong scalar field background
- However, note that we obtained this result using **Feynman** diagrammatic technique, i.e. we neglected the back-reaction and photons' production

Loop corrections

- To account for the back-reaction and strong vacuum perturbation we use **Schwinger-Keldysh** diagrammatic technique which correctly describes non-stationary situations
- The state of the theory is encoded in the Keldysh bosonic propagator and trace of the Keldysh fermionic propagator:

$$D^K(t_1, t_2; p) = \left(\langle \alpha_p^+ \alpha_p \rangle + \frac{1}{2} \right) f_p(t_1) f_p^*(t_2) + \langle \alpha_p \alpha_{-p} \rangle f_p(t_1) f_p(t_2) + h.c.,$$

$$\begin{aligned} \operatorname{tr} G_{ab}^K(t_1, t_2; p) = \left(\langle a_p^+ a_p \rangle - \frac{1}{2} \right) \left(\psi_{p1}^2 \psi_{p2}^{2*} - \psi_{p1}^1 \psi_{p2}^{1*} \right) - \\ - \langle a_p b_{-p} \rangle \left(\psi_{p1}^1 \psi_{p2}^2 + \psi_{p1}^2 \psi_{p2}^1 \right) + \left(a \to b,\ p \to -p,\ h.c. \right), \end{aligned}$$

where average is done over an arbitrary state which respects spatial transnational invariance

Loop corrections

At the one-loop level we find that:

- Number of bosons, i.e. boson level density, grows linearly: $n_p(t) = \langle \alpha_p^+ \alpha_p \rangle \sim \lambda^2 t$
- Boson anomalous quantum average do not receive growing with time corrections: $\kappa_p(t) = \langle \alpha_p \alpha_{-p} \rangle \sim \mathcal{O}(\lambda^2 t^0)$
- Fermion anomalous quantum average, number of fermions and antifermions grow logarythmically: $\langle a_p b_{-p} \rangle \sim \langle a_p^+ a_p \rangle \sim \langle b_{-p}^+ b_{-p} \rangle \sim \lambda^2 \log t$
- Thus, loop corrections are comparable with the tree-level current

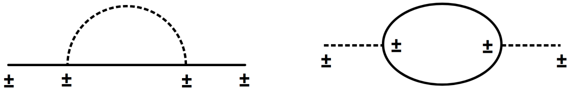

Figure: One loop corrections to the fermion and boson two-point functions in '$\pm$' notation

Conclusion

- We considered 2D Yukawa model on the $\phi_{cl} = Et$ background which is a simplified model of the Schwinger pair creation
- We calculated the fermionic current and showed that it grows with time
- We generalized this result to an arbitrary scalar field backgrounds using effective action
- We calculated one-loop corrections to the level densities and anomalous quantum averages of bosons and fermions using Keldysh diagrammatic technique
- We showed that loop corrections are comparable with the tree-level expressions
- In the near future we plan to solve the DS equation and sum up the leading corrections from all loops