Non-commutativity for Closed String on H-monopole background

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Abstract

Discussion of calculation of non-commutativity for closed string on H-monopole background.

1. H-monopole

Our aim is to calculate non-commutativity (similar to [1]) for the closed string on H-monopole background. H-monopole is once smeared NS5-brane along one of its localization coordinates [2]. ***H-monopole*** background looks like:

$$\begin{cases} ds^2 = -dt^2 + d\vec{y}_5^2 + H(d\vec{x}_3^2 + dz^2), \\ B_2 = A_i dx^i \wedge dz, \qquad i = \overline{1,3}, \\ e^{-2(\varphi-\varphi_0)} = H^{-1}, \end{cases}$$

here z - is compact coordinate with radius R_z, from NS5 smearing process we get $H = 1 + \frac{m}{2R_z r}$, $r = |x^i|$ (we use here and everywhere bellow notation $i, j, k = \overline{1,3}$). Finally the 1-form A obeys:

$$2\partial_{[i}A_{j]} = \epsilon_{ijk}\partial_k H \qquad \Longleftrightarrow \qquad \epsilon_{kij}\partial_{[i}A_{j]} = \partial_k H \tag{1}$$

To solve it, we divide space into two parts $x_3 > 0$ - North pole (N), $x_3 < 0$ - South pole (S), for each part we find solutions in spherical coordinates:

$$A^N_\varphi = 2g(1 - cos\theta), \qquad A^S_\varphi = -2g(1 + cos\theta), \tag{2}$$

all the other components are zero. We can see, that solutions in different maps are linked by ***gauge transformation***:

$$A^S_\varphi - A^S_\varphi = 4g = \partial_\varphi \lambda, \qquad \lambda = 4g\varphi. \tag{3}$$

2. Monodromy

For pure defined backgrounds , we make some coordinate identification to make background globally defined. Such a transformation is called ***monodromy*** and is given as matrix P^N_M (wich belongs to a group $O(d,d)$ for DFT solutions), it transforms generalised metric as:

$$\tilde{\mathcal{H}}_{MN} = P_M{}^R P_N{}^Q \mathcal{H}_{RQ}, \tag{4}$$

where generalised metric is:

$$\mathcal{H}_{MN} = \begin{pmatrix} G - BG^{-1}B & BG^{-1} \\ -G^{-1}B & G^{-1} \end{pmatrix} = \begin{pmatrix} \mathbb{1} & B \\ 0 & \mathbb{1} \end{pmatrix} \begin{pmatrix} G & 0 \\ 0 & G^{-1} \end{pmatrix} \begin{pmatrix} \mathbb{1} & 0 \\ B^T & \mathbb{1} \end{pmatrix}, \; B^T = -B. \tag{5}$$

Closed string on background with monodromy should have following periodicity (boundary) conditions:

$$X^M(\sigma + 2\pi) = P^M{}_N X^N(\sigma) + 2\pi w^M. \tag{6}$$

H-monopole is ill defined in another way. It is defined in terms of two solutions lying in two different intersecting maps, which cover the entire space. These two solutions are connected by gauge transformation (3). For H-monopole we can write metric, Kalb-Ramond field and gauge transformation $\Lambda = B^N_{ij} - B^S_{ij}$ explicitly (tab.1). Using (5) we will get $\mathcal{H}^N_{MN}$, $\mathcal{H}^S_{MN}$ metrics for both maps. From (5) we can see connection:

$$\mathcal{H}^N_{MN} = U_M{}^R U_N{}^Q \mathcal{H}^S_{RQ}, \qquad U_M{}^R = \begin{pmatrix} \mathbb{1} & \Lambda \\ 0 & \mathbb{1} \end{pmatrix}. \tag{7}$$

Tab.1. Metric, Kalb-Ramond field and gauge transformation for H-monopole in spherical coordinates.

	G_{ij}	B_{ij}	$\Lambda = B^N_{ij} - B^S_{ij}$
H	$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & r^2 & 0 & 0 \\ 0 & 0 & r^2sin^2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_\varphi \\ 0 & 0 & -A_\varphi & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4g \\ 0 & 0 & -4g & 0 \end{pmatrix}$

We interpret this transformation as analogue of monodromy for H-monopole. To summarize, we will disscuss two types of monodromies: 1) monodromy as transformation (6) making background globally defined (we denote it as $P_M{}^N$); 2) monodromy as transformation gluing background in different maps (we denote it as $U_M{}^N$ and call ***gauge monodromy***)

3. Equations of Motion

From Double Field Theory action:

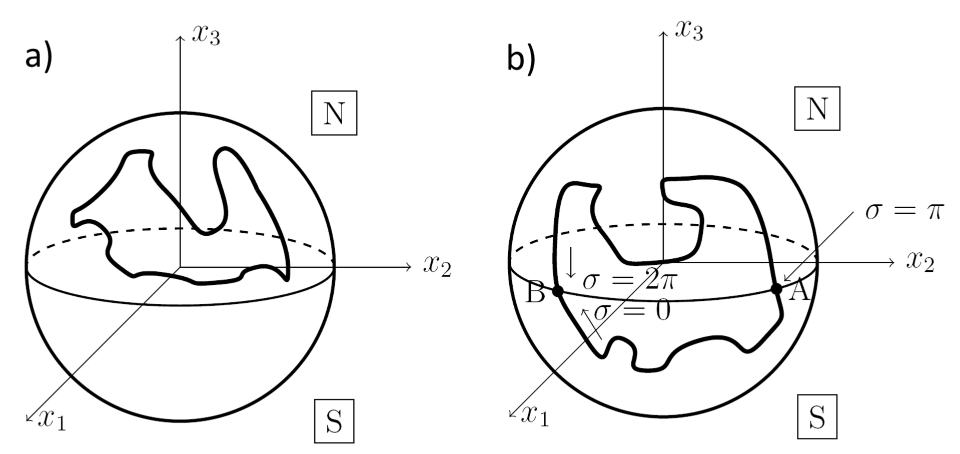
$$S_{DFT} = \int d\tau d\sigma \left(\frac{1}{2}\dot{X}^M \eta_{MN} X'^N - \frac{1}{2} X'^M \mathcal{H}_{MN} X'^N \right) + \pi \int d\tau w^M \eta_{MN} P^N{}_P \dot{X}^P(\sigma = 0), \tag{8}$$

where $\eta_{MN} = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix}$ and $P^M{}_N$ is monodromy from (6), we get general form of EOM:

$$(-\eta_{MN}\dot{X}^N + \mathcal{H}_{MN}X'^N)' + \delta(\sigma - 2\pi)(\frac{1}{2}\eta_{MN}\dot{X}^N(2\pi) - \mathcal{H}_{MN}X'^N(2\pi)) + \\ + \delta(\sigma)(\mathcal{H}_{MN}X'^N(0) - \frac{1}{2}\eta_{MN}\dot{X}^N(0) - \pi\dot{w}^P \eta_{PQ} P^Q{}_M) = \frac{1}{2} X'^P (\partial_M \mathcal{H}_{PQ}) X'^Q. \tag{9}$$

There are two essential cases of string location:

Pic.1. a) Closed string entirely lying in one map; b) Closed string lying in two maps.



For ***closed string in one map*** (Pic.1.a.) we don't have monodromy for period. conditions (6):

$$X^M(\sigma + 2\pi) = X^M(\sigma) + 2\pi w^M \implies P^N_M = \mathbb{1}. \tag{10}$$

Interesting case is ***closed string lying in two maps***. We look for solutions of (9) defined in two maps (see Pic.1.b.). We glue N and S solutions in points A and B by gauge monodromy:

$$X^M(\sigma = \pi) = U^M{}_N Y^N(\sigma = \pi), \qquad X^M(\sigma = 2\pi) = U^M{}_N Y^N(\sigma = 0) + 2\pi w^M, \tag{11a}$$

where X - North solution, Y - South solution (here we assumed and will assume it further, that string crosses chart boundary twice). This case we study further.

4. Mode expansions

From (11) we have following ***nontrivial boundary conditions***:

$$\tilde{\Phi}_N(\sigma = \pi + \epsilon) = \tilde{\Phi}_S(\sigma = \pi + \epsilon) - 4gZ_S(\sigma = \pi + \epsilon), \tag{12a}$$

$$\tilde{Z}_N(\sigma = \pi + \epsilon) = \tilde{Z}_S(\sigma = \pi + \epsilon) + 4g\Phi_S(\sigma = \pi + \epsilon), \tag{12b}$$

$$\tilde{\Phi}_N(\sigma = 2\pi + \epsilon) = \tilde{\Phi}_S(\sigma = 0 + \epsilon) - 4gZ_S(\sigma = 0 + \epsilon) + 2\pi w^{\tilde{\varphi}}, \tag{12c}$$

$$\tilde{Z}_N(\sigma = 2\pi + \epsilon) = \tilde{Z}_S(\sigma = 0 + \epsilon) + 4g\Phi_S(\sigma = 0 + \epsilon) + 2\pi w^{\tilde{z}}, \tag{12d}$$

$$Z_N(\sigma = \pi + \epsilon) = Z_S(\sigma = \pi + \epsilon), \qquad Z_N(\sigma = 2\pi + \epsilon) = Z_S(\sigma = 0 + \epsilon) + 2\pi w^z, \tag{12e}$$

$$\Phi_N(\sigma = \pi + \epsilon) = \Phi_S(\sigma = \pi + \epsilon), \qquad \Phi_N(\sigma = 2\pi + \epsilon) = \Phi_S(\sigma = 0 + \epsilon) + 2\pi w^\varphi, \tag{12f}$$

$\epsilon \in (-\delta, \delta), \delta > 0$, here we use indices S and N for South and North solutions. Z, R, Θ, $\tilde{R}$ and $\tilde{\Theta}$ are glued trivially (they are described by same functions in two maps). We write mode expansions, which satisfy (12):

$$\Phi_S(\sigma) = \varphi + w^\varphi \sigma + \sum_{n \neq 0} \alpha^\varphi_n e^{in\sigma} = \Phi_N(\sigma), \qquad Z_S(\sigma) = z + w^z\sigma + \sum_{n \neq 0} \alpha^z_n e^{in\sigma} = Z_N(\sigma), \tag{13a}$$

$$\tilde{\Phi}_S(\sigma) = \tilde{\varphi} + w^{\tilde{\varphi}}\sigma + \sum_{n \neq 0} \alpha^{\tilde{\varphi}}_n e^{in\sigma}, \qquad \tilde{Z}_S(\sigma) = \tilde{z} + w^{\tilde{z}}\sigma + \sum_{n \neq 0} \alpha^{\tilde{z}}_n e^{in\sigma}, \tag{13b}$$

$$\tilde{\Phi}_N(\sigma) = (\tilde{\varphi} - 8gw^z\pi - 4gz) + (w^{\tilde{\varphi}} + 4gw^z)\sigma + \sum_{n \neq 0} (\alpha^{\tilde{\varphi}}_n - 4g\alpha^z_n) e^{in\sigma}, \tag{13c}$$

$$\tilde{Z}_N(\sigma) = (\tilde{z} + 8gw^\varphi\pi + 4g\varphi) + (w^{\tilde{z}} - 4gw^\varphi)\sigma + \sum_{n \neq 0} (\alpha^{\tilde{z}}_n + 4g\alpha^\varphi_n) e^{in\sigma}. \tag{13d}$$

5. Dirac bracket calculation

We are interested in ***nontrivial commutators*** for string coordinates. They arise from replacement of Poisson bracket by ***Dirac bracket***:

$$\{f, g\}_{Dirac} = \{f, g\} - \{f, C_a\}\{C_a, C_b\}^{-1}\{C_b, g\}, \qquad C_a = \Pi_a - \frac{\partial L}{\partial \dot{a}}, \tag{14}$$

where Poisson bracket defined as $\{\chi, \Pi_\chi\} = 1$, and zero for oher cases. Through strait forward calculations for DFT action (8) with modes (13) we find constraints and their commutators, for example:

$$\{C_\varphi, C_{\tilde{\varphi}}\} = -\int_0^{2\pi} d\sigma \frac{1}{2} \Big\{ R\tilde{R}sin(\Theta)cos(\tilde{\Theta})cos(\tilde{\Phi} - \Phi)\tilde{\Theta}' + R\,sin(\Theta)sin(\tilde{\Theta})cos(\tilde{\Phi} - \Phi)\tilde{R}' + \\ - R\tilde{R}cos(\Theta)sin(\tilde{\Theta})cos(\Phi - \tilde{\Phi})\Theta' - \tilde{R}sin(\Theta)sin(\tilde{\Theta})cos(\Phi - \tilde{\Phi})R' \Big\} + \\ - \int_0^{\pi} d\sigma \frac{1}{2} R\tilde{R}sin(\Theta)sin(\tilde{\Theta})sin(\Phi - \tilde{\Phi})(w^{\tilde{\varphi}} + \sum_{n \neq 0} in\alpha^{\tilde{\varphi}}_n e^{in\sigma}) + \\ - \int_\pi^{2\pi} d\sigma \frac{1}{2} R\tilde{R}sin(\Theta)sin(\tilde{\Theta})sin(\Phi - \tilde{\Phi})(w^{\tilde{\varphi}} + 4gw^z + \sum_{n \neq 0} in(\alpha^{\tilde{\varphi}}_n - 4g\alpha^z_n)e^{in\sigma}) + \\ - \int_0^{2\pi} d\sigma \frac{1}{2} R\tilde{R}sin(\Theta)sin(\tilde{\Theta})sin(\Phi - \tilde{\Phi})(w^\varphi + \sum_{n \neq 0} in\alpha^\varphi_n e^{in\sigma}) \\ + \pi(R\tilde{R}sin(\Theta)sin(\tilde{\Theta})cos(\Phi - \tilde{\Phi})w^{\tilde{\varphi}} + R\tilde{R}sin(\Theta)cos(\tilde{\Theta})cos(\tilde{\Phi} - \Phi)w^{\tilde{\theta}} + R\,sin(\Theta)sin(\tilde{\Theta})cos(\tilde{\Phi} - \Phi)w^{\tilde{r}}) + \\ - \pi(R\tilde{R}sin(\Theta)sin(\tilde{\Theta})cos(\Phi - \tilde{\Phi})w^\varphi + R\tilde{R}cos(\Theta)sin(\tilde{\Theta})cos(\Phi - \tilde{\Phi})w^\theta + \tilde{R}sin(\Theta)sin(\tilde{\Theta})cos(\Phi - \tilde{\Phi})w^r) \tag{15}$$

6. Discussion - Non-commutativity?

On the stage of Dirac bracket calculation, computations become bulky and doesn't allow yet to obtain the final result - non-trivial or trivial Dirac bracket. Presently, to overcome this problem we are trying to find more tricky way to write constraints, for example to isolate total derivatives.

References

[1] Chris D. A. Blair. Non-commutativity and non-associativity of the doubled string in non-geometric backgrounds. URL: arXiv:1405.2283.

[2] Ilya Bakhmatov, Axel Kleinschmidt, and Edvard T. Musaev. Non-geometric branes are dft monopoles. URL: arXiv:1607.05450, doi:10.1007/JHEP10(2016)076.