On inhomogeneous states in CP(N-1) and O(N) sigma models

Arsenii Pikalov

Institute for Theoretical and Experimental Physics

Moscow Institute of Physics and Technology

25th February, 2019

Structure of the talks

- Sigma models: $\mathbb{CP}^{N-1}$ and $O(N)$.
- Large N approximation and gap equation
- Solitonic solution
- Periodic solution
- Discussion

The talk is based on preprint A. Gorsky, A. Pikalov, A. Vainshtein, "On instability of ground states in 2D CP(N-1) and O(N) models at large N" (arXiv:1811.05449).

The definitions of the models

Work in $1+1$ **Minkowski space-time**
Action for $\mathbb{CP}^{N-1}$ model:

$$S = \int d^2x \left(|D_\mu n^a|^2 - \lambda(|n^a|^2 - r) \right),$$

$$D_\mu = \partial_\mu - iA_\mu, \; a = 1, \ldots, N.$$

Action for $O(N)$ model:

$$S = \frac{1}{2} \int d^2x \left((\partial_\mu n^a)^2 - \lambda \left((n^a)^2 - r \right) \right).$$

Motivation

Why to be interested in the $\mathbb{CP}^{N-1}$ model?

- Similarity to the Yang-Mills theory
- Effective description of non-Abelian strings
- 2d/4d correspondence in SUSY theories
- Compactification of Yang-Mills on a torus (Yamazaki, Yonekura)
- Summation of perturbation theory series (Dunne, Unsal)

Large N approximation

For simplicity consider $O(N)$ case.
The action is quadratic in n, so they can be integrated

$$\int Dn\, D\lambda e^{iS[n^a,\lambda]} = \exp(iS_{eff})$$

$$S_{eff} = \frac{iN}{2} tr \log(-\partial^2 - \lambda) + \frac{1}{2}\int d^2x \left((\partial_\mu n)^2 - \lambda\left((n)^2 - r\right)\right)$$

Here we separated "classic" and "quantum" parts $n^a = n_q^a + nl^a$, $(l^a)^2 = 1$.
If $N \gg 1$ saddle point approximation is valid.

Homogeneous solution

Look for time-independent solutions with $\lambda = \lambda(x);\ n = n(x)$. The equations are

$$n^2 = r - N\sum_n \frac{|f_n(x)|^2}{2E_n};\ (-\partial_x^2 + \lambda(x))f_n(x) = E_n^2 f_n(x);$$

$$(-\partial_x^2 + \lambda(x))n(x) = 0.$$

Standard solution:

$$\lambda = const = m^2;\ n = 0.$$

Mass is generated via dimensional transmutation.

Solitonic solution

Another solution (Nitta and Yoshii, 2017):

$$\lambda = m^2 \left(1 - \frac{2}{\cosh^2 mx}\right);\ n^2 = \frac{N}{2\pi}\frac{1}{\cosh^2 mx}.$$

$$f_k(x) = \frac{ik - m\tanh mx}{\sqrt{m^2 + k^2}} e^{ikx};\ E_k^2 = k^2 + m^2$$

$$|f_k|^2 = 1 - \frac{m^2}{m^2 + k^2}\frac{1}{\cosh^2 mx}.$$

Properties of the solution

The energy is lower than the energy of homogeneous state.
How does the ground state of the model look like?

$$E = -\frac{Nm}{\pi}.$$

Calculation was performed by two independent ways

- calculation of effective action value
- calculation of the average of energy-momentum tensor in inhomogeneous background

There are zero modes corresponding to the rotations of the n field in the internal space. Integration over them yields the volume of S_{N-1}

Gross-Neveu model

$$S = \frac{1}{2}\int d^2x \left(\bar{\psi}^a(i\hat{\partial} - \sigma)\psi^a - r\sigma^2\right)$$

It is fermionic part of SUSY $O(N)$ model.
Homogeneous solutions are

$$\sigma = \pm m \sim \langle \bar{\psi}\psi \rangle$$

There are also kink solutions

$$\sigma = \pm m \tanh mx.$$

They correspond to the $O(N)$ soliton:

$$\lambda = \sigma^2 - \partial_x \sigma.$$

Periodic solution

Similarly to Gross-Neveu case look for solution

$$\lambda(x) = m_1^2\nu(2sn^2(m_1x;\, \nu) - 1);\, n \sim dn(m_1x;\, \nu).$$

Properties

- Corresponds to the ground state of Gross-Neveu model at finite density
- Ill-defined in $O(N)$ case on $\mathbb{R}^2$ due to IR divergences
- Formally, energy is lower than in homogeneous case

About ground state

- Homogeneous solution is not the ground state at large N
- Periodic solution is not well-behaved on the whole plane
- Euclidean correlators in $O(N)$ model decay exponentially (Kopper, 1998)

Open questions

- Difference between $O(N)$ and $\mathbb{CP}^{N-1}$
- Role of the gauge field
- SUSY generalizations
- Connection with classical solutions

Thank you for attention!