Black hole basics

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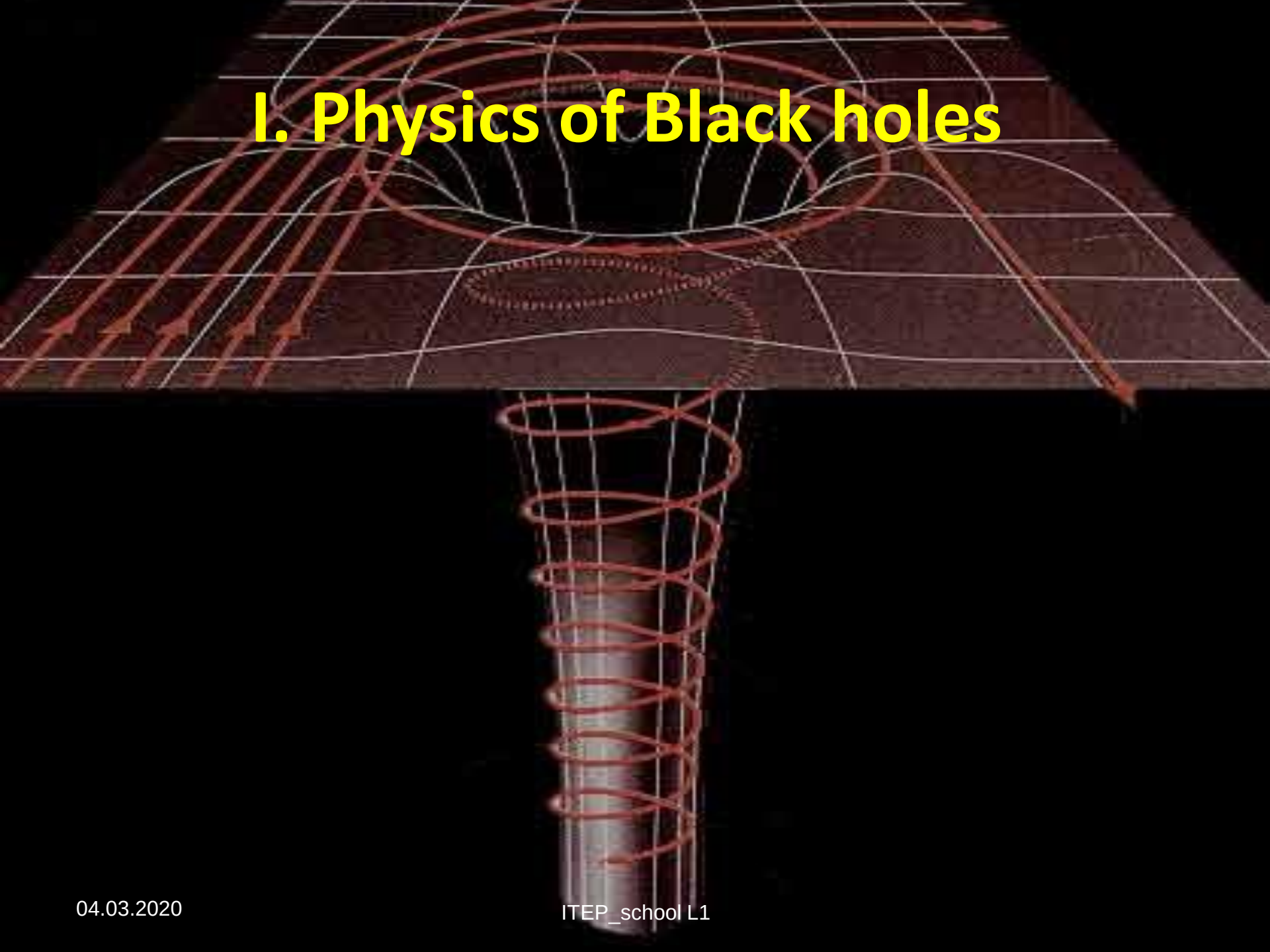
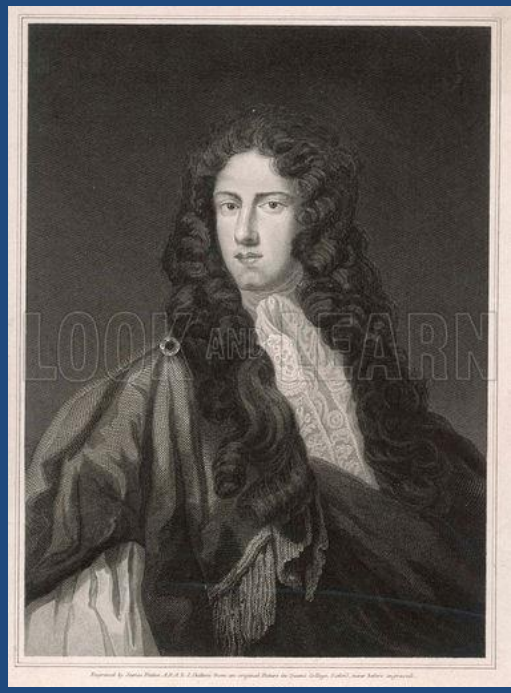
I. Physics of Black holes



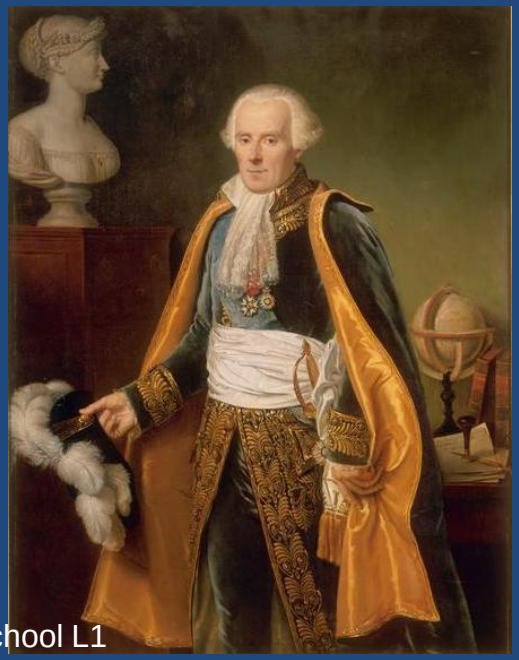
History, 18-19 century

- 1783, John Mitchell, Phil. Trans. Roy. Soc. London, "dark stars"

$$R_g = \frac{2GM}{c^2} \approx 3km\left(\frac{M}{M_\odot}\right)$$



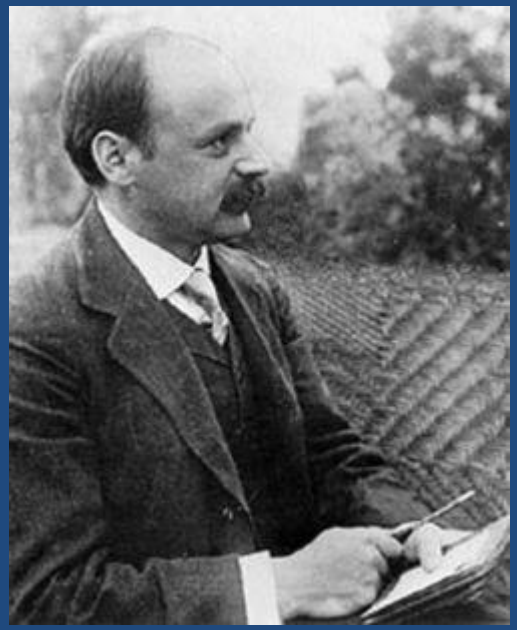
- 1796, P.-S. Laplace, *Exposition du système du Monde* (1st and 2d eds)



- In Newtonian physics , a body with $< R_g$ cannot communicate with outer world by light, but can have a magnetic field, for example.
- There are no “event horizon” in Newtonian physics, one can construct a static star with $R<R_g$

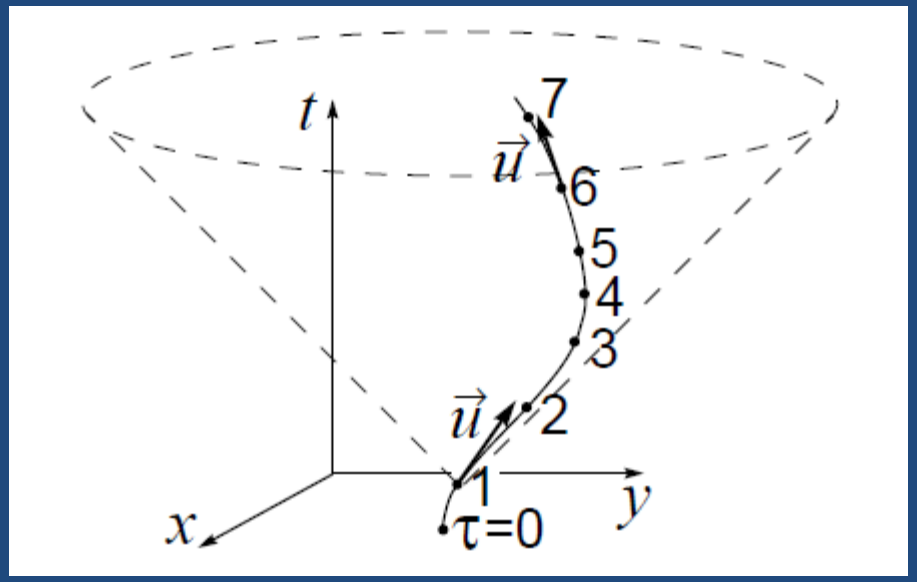
History, after Einstein

- 1915, A.Einstein, GR
- 1916, K. Schwarzschild, metrics of spherically symmetric spacetime outside a massive body
- 1924, A.Eddington, coordinate singularity
- 1939, Oppenheimer, Snyder, grav. collapse, ‘frozen stars’
- 1958, Finkelstein coordinate transformation
- 1963, Kerr solution
- 1960s, Penrose, Hawking, Carter… -- black hole theorems
- 1968, J.A Wheeler, term ‘black hole’
- 1970s, massive discovery of black hole candidates in X-ray binaries
- 2015- massive detections of binary stellar-mass BH by LIGO/Virgo GW detectors



$$ds^2 = \left(1 - \frac{r_s}{r}\right) c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{r_s}{r}\right)} - r^2 \left(\sin^2 \theta d\varphi^2 + d\theta^2\right),$$

Mapping spacetime



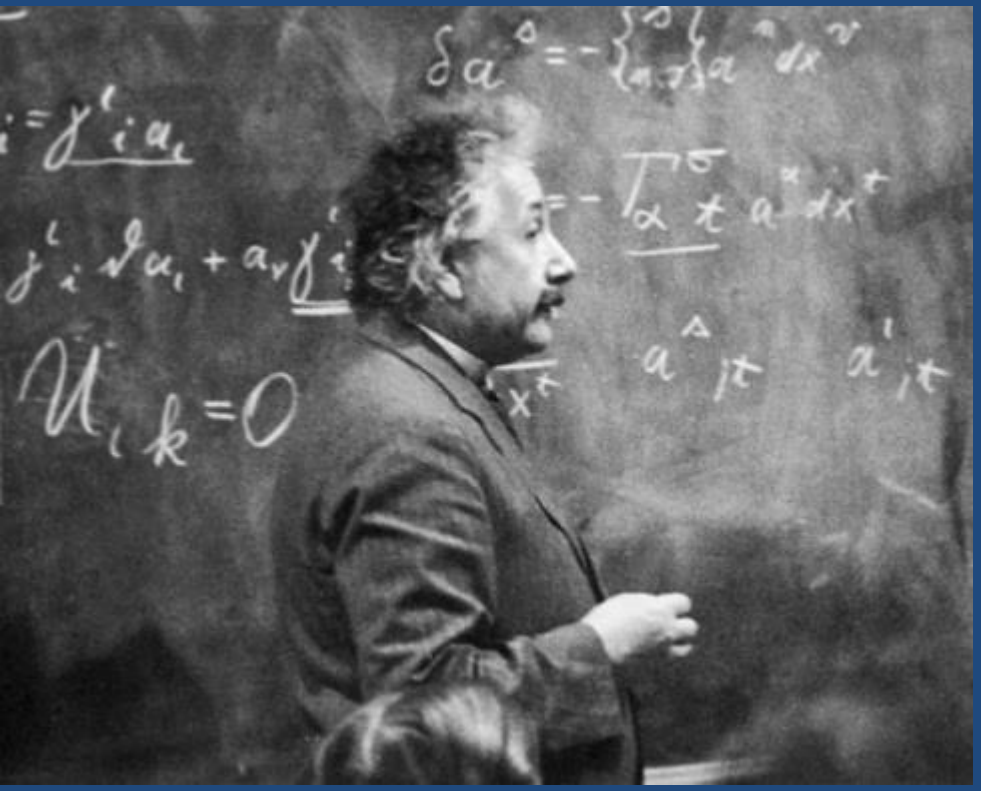
$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

$$ds^2 = \eta_{ik} dx^i dx^k,$$

$$\eta_{ik} = diag(-1,1,1,1)$$

- **Special relativity (no gravity): Minkowski metric (Lorentz frames)**
- **Real particle's world line lies inside light cone (timelike geodesics), $ds^2=-d\tau^2<0$**
- **Light cone is formed by null geodesics, $ds^2=0$**

General Relativity



- A. Einstein, 1912-1915
- Gravitation = curvature of spacetime

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$R_{\alpha\beta\gamma\delta} = \frac{1}{2}(g_{\alpha\delta,\beta\gamma} + g_{\beta\gamma,\alpha\delta} - g_{\alpha\gamma,\beta\delta} - g_{\beta\delta,\alpha\gamma}) \quad \text{in a local Lorentz frame.}$$

$$R_{\alpha\beta} \equiv R^{\mu}{}_{\alpha\mu\beta}$$

$$R \equiv R^{\alpha}{}_{\alpha}$$

Geometrized units

$$G = c = 1$$

$$r = GM / c^2 = M$$

$$t = GM / c^3 = M$$

Quantity	Conventional Units	Geometrized Units
speed of light, c	2.998×10^8 m sec^{-1}	one
Newton's gravitation constant, G	6.674×10^{-11}m^3 kg^{-1} sec^{-2}	one
G/c^2	7.426×10^{-28} m kg^{-1}	one
c^5/G	3.628×10^{52} W	one
Planck's reduced constant $\hbar$	1.055×10^{-34} kg m^{2}s^{-1}	$(1.616 \times 10^{-35}$ m$)^2$
sun's mass, $M_\odot$	1.989×10^{30} kg	1.4766 km
sun's radius, $R_\odot$	6.960×10^8 m	6.960×10^8 m
earth's mass, $M_\oplus$	5.976×10^{24} kg	4.435 mm
earth's radius, $R_\oplus$	6.371×10^6 m	6.371×10^6 m
Hubble constant H_o	70.5 km sec^{-1}Mpc^{-1}	$(12.9 \times 10^9$ lt yr$)^{-1}$
density to close universe, $\rho_{\rm crit}$	1.08×10^{-26} kg m^{-3}	8.0×10^{-54} m^{-2}

Schwarzschild metric

- 13 January 1916, letter to A.Einstein from WWI Russian front
- Empty space T=0, M is integration constant

$$\boxed{ds^2 = -(1-2M/r)dt^2 + \frac{dr^2}{(1-2M/r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)}$$

- t=const, r=const ➔ 2-sphere

$$^{(2)}ds^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- r is not a distance

to the center!!!!

$$\boxed{\text{circumference} = 2\pi r\ , \quad \text{area} = 4\pi r^2}$$

- $r/M \to \infty$: $$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$
- Asymptotically flat spacetime!
- To first order in M/r:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 + \frac{2M}{r}\right) dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- Linearized gravity $$ds^2 = -(1 + 2\Phi)dt^2 + (\delta_{jk} + h_{jk})dx^j dx^k + 2h_{tj}dt\, dx^j$$
- $\Phi = -M/r$ is Newtonian potential ➔ *M* is the mass of the central body
- @ $r>2M$ t plays the role of a time coordinate ➔ outside grav. radius the metric is static

Birkhoff ‘s theorem (1923)

- Schwarzschild geometry is unique spherically symmetric solution of Einstein vacuum field equation G=0
- Important implication: static star in vacuum (T=0), at r=R circumference $2\pi R$ ➔ since any motion of real particles on the surface is timelike ($d\tau^2=-ds^2=-g_{00}dt^2>0$)➔ **a static star with total mass-energy M can never have circumference smaller than *$2\pi R=4\pi M$***

Embedding diagrams

- Embedding of a curved manifold in a flat space of higher dimension: N≥n(n+1)/2. Doable and instructive for n=2 ➔ N=3.
- Example: static star, r=const, θ=π/2

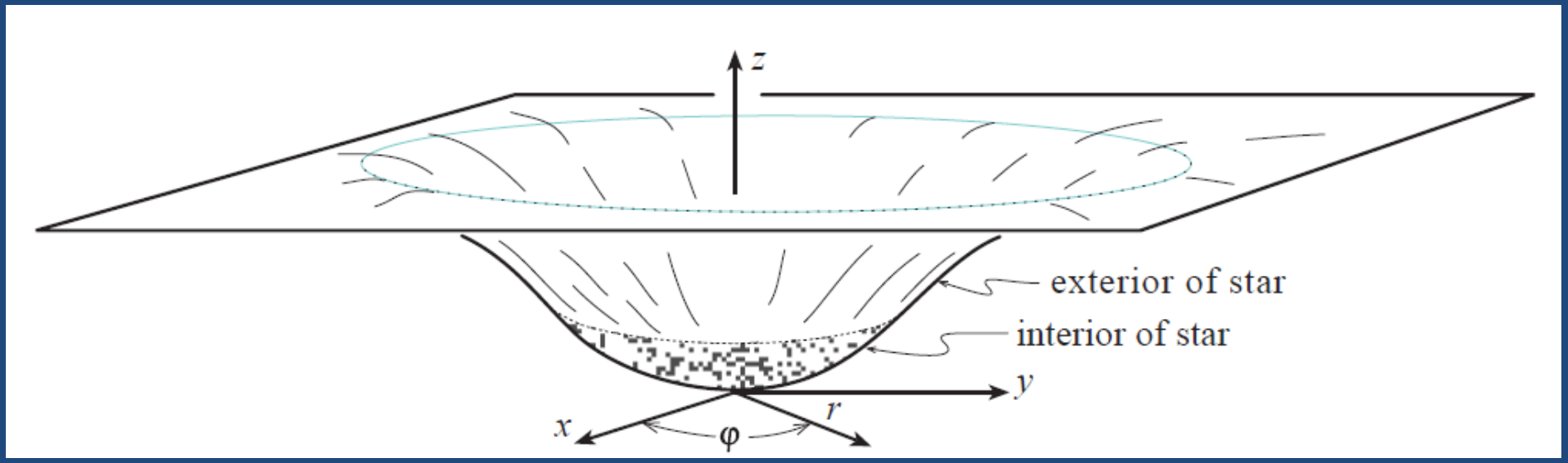
$$^{(2)}ds^2 = \frac{dr^2}{1-2m/r} + r^2 d\phi^2$$

$$^{(3)}ds^2 = dr^2 + dz^2 + r d\phi^2$$

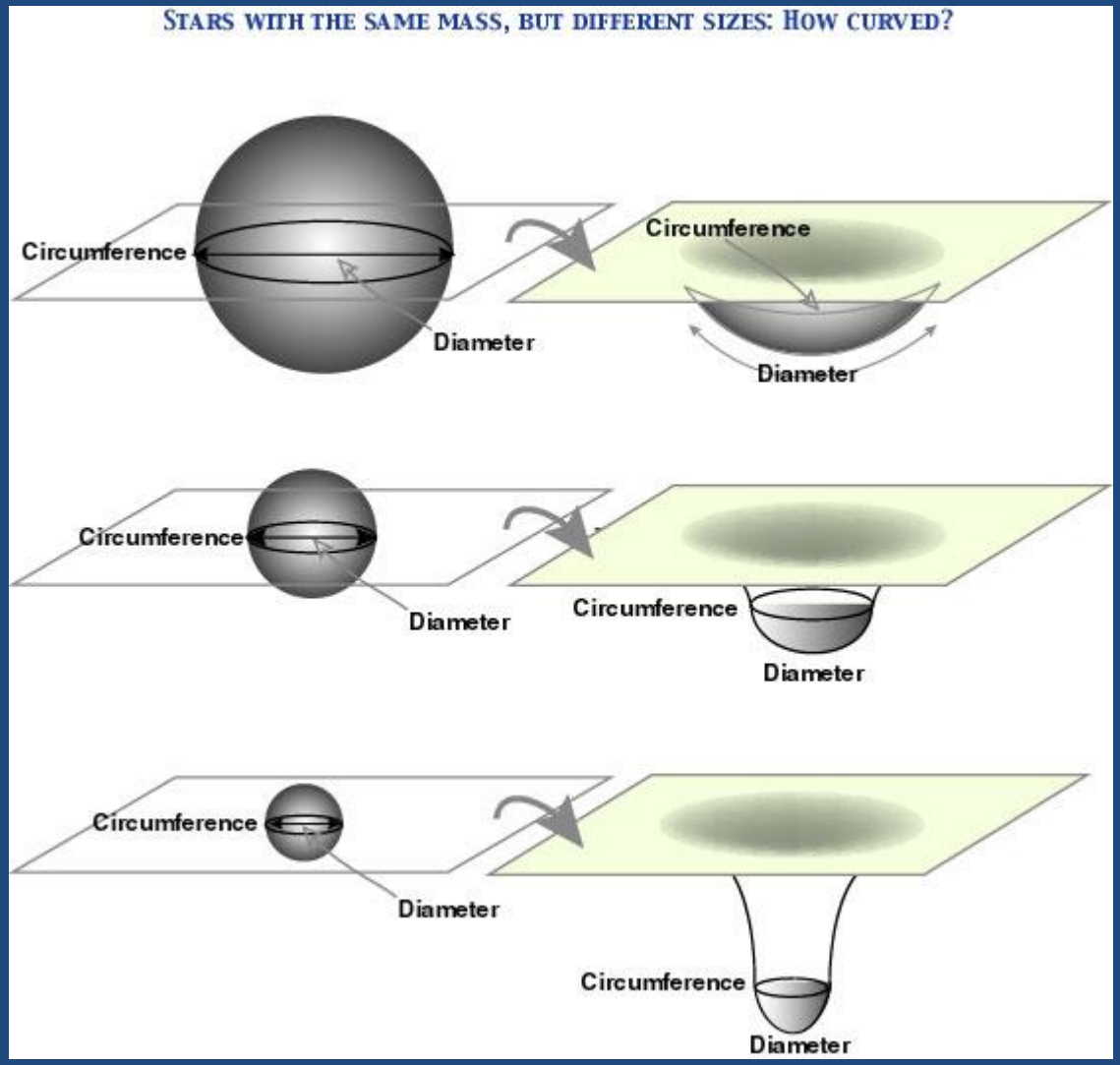
$$^{(2)}ds^2 = [1 + (dz/dr)^2]dr^2 + r^2 d\phi^2$$

$$\frac{dz}{dr} = \left(\frac{1}{1-2m/r} - 1\right)^{\frac{1}{2}}$$

$$\boxed{z = \int_0^r \frac{dr}{[(r/2m) - 1]^{\frac{1}{2}}}}$$



- Circumference increases more slowly than the proper radial distance growth $l = \int_0^r (1 - 2m/r)^{-\frac{1}{2}} dr$



- $r/l \sim 1 - r_g/r$ ➔ 1.5 mm for Earth, 10% for NS

Symmetries

$$Killing\ \ vectors: \quad \boldsymbol{\xi}\cdot\mathbf{u} = const$$

$$\boldsymbol{\xi} = (1,0,0,0) \quad t-independence$$

$$\eta = (0,0,0,1) \quad \varphi-independence$$

Example 1: grav. redshift

$$\boldsymbol{\xi}\cdot\mathbf{u} = const \Leftrightarrow \boldsymbol{\xi}\cdot\mathbf{p} = const$$

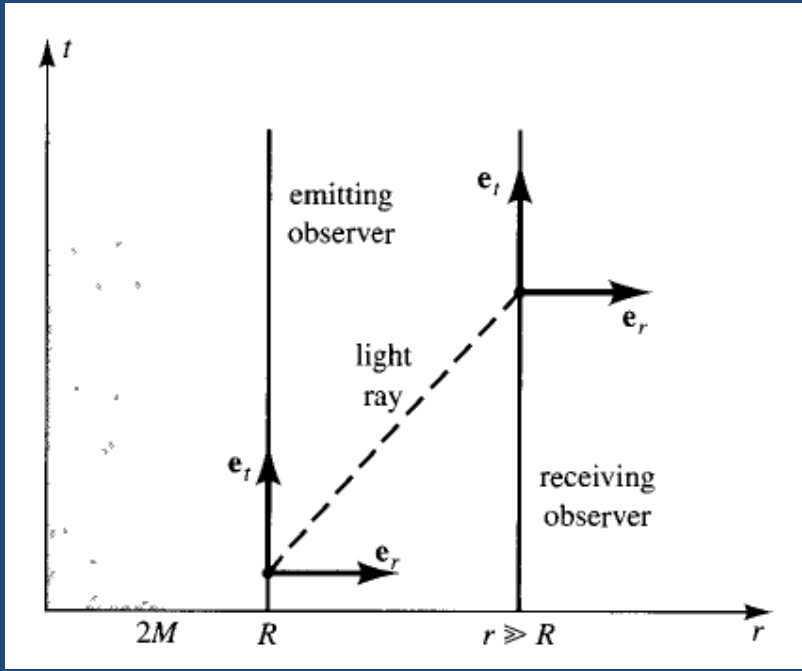
$$E = \hbar\omega = p^0 = -\mathbf{p}\cdot\mathbf{e}_0 = -\mathbf{p}\cdot\mathbf{u}_{obs}$$

$$u^i_{obs} = 0, \quad \mathbf{u}_{obs}\cdot\mathbf{u}_{obs} = -1 \Rightarrow$$

$$g_{tt}(u^t_{obs})^2 = -1, \quad u^\alpha_{obs} = [-g_{tt}]^{-1/2}\xi^\alpha$$

$$\left.\begin{aligned} \hbar\omega_{em} &= [-g_{tt}]^{-1/2}(-\boldsymbol{\xi}\cdot\mathbf{p})\big|_R \\ \hbar\omega_\infty &= [-g_{tt}]^{-1/2}(-\boldsymbol{\xi}\cdot\mathbf{p})\big|_\infty \end{aligned}\right\} \Rightarrow$$

$$\omega_\infty = \omega_{em}\sqrt{1-\frac{2M}{R}}$$



Example 2: Orbits

$$t: \quad e = -\boldsymbol{\xi}\cdot\mathbf{u} = \left(1-\frac{2M}{r}\right)\frac{dt}{d\tau}$$

Energy per unit rest mass

$$\varphi: \quad l = -\boldsymbol{\eta}\cdot\mathbf{u} = r^2\sin^2\theta\frac{d\varphi}{d\tau}$$

Ang. mom. per unit rest mass

$$\mathbf{u}\cdot\mathbf{u} = g_{\alpha\beta}u^\alpha u^\beta = -1, \quad \sin\theta = 1$$

$$\underbrace{\frac{e^2-1}{2}}_{\mathrm{E}} = \frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + \frac{1}{2}\left[\left(1-\frac{2M}{r}\right)\left(1+\frac{l^2}{r^2}\right)-1\right]$$

Integral of energy

$$V_{eff}(r) \equiv \frac{1}{2}\left[\left(1-\frac{2M}{r}\right)\left(1+\frac{l^2}{r^2}\right)-1\right] = -\frac{M}{r}+\frac{l^2}{2r^2}-\frac{Ml^2}{r^3}$$

Effective potential

$$\mathrm{E} = \frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + V_{eff}(r)$$

Innermost stable circular orbits

$$r_{\substack{\min\\ \max}} = \frac{l^2}{2M}\left[1 \mp \sqrt{1-12\left(\frac{M}{l}\right)^2}\right]$$

$$ISCO: \;\; l/M = \sqrt{12}$$

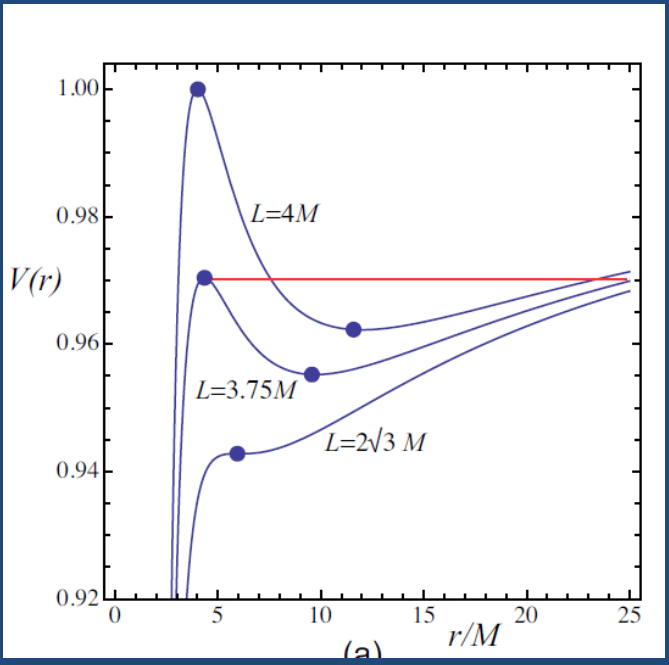
$$r_{ISCO} = \frac{l^2}{2M} = 6M$$

$$\frac{d\varphi}{dt} = \Omega, \;\; \Omega^2(r) = \frac{M}{r^3}$$

$$\frac{d\varphi}{dt} = \frac{d\varphi/d\tau}{dt/d\tau} = \frac{\Omega}{u^t} = \frac{1}{r^2}\left(1-\frac{2M}{r}\right)\frac{l}{e}$$

$$u^\alpha = u^t(1,0,0,\Omega)$$

$$\mathbf{u}\cdot\mathbf{u} = -1 \Rightarrow u^t = \left(1-\frac{3M}{r}\right)^{-1/2}$$



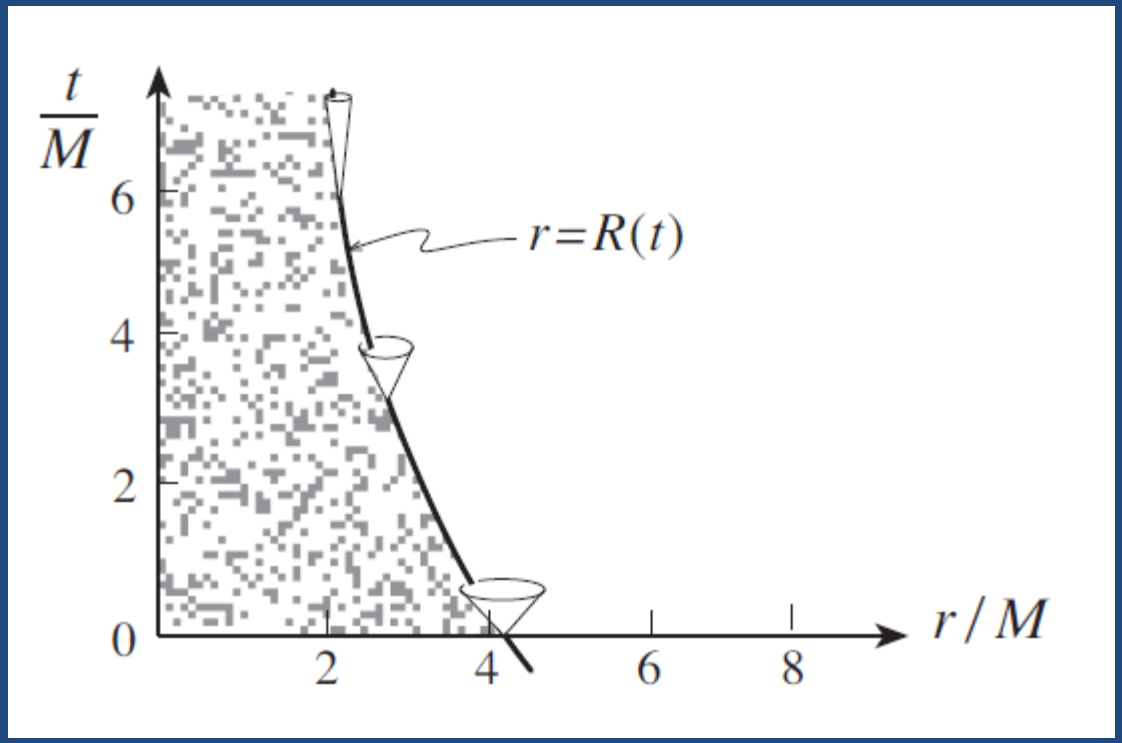
$$\eta = \frac{m - e\cdot m}{m} = 1 - e$$

$$@ISCO: \eta = 1 - 2\sqrt{2}/3 \approx 0.06$$

$$(cf: thermonuclear\ burning\ \eta_{nucl} \approx 0.0067)$$

Collapse to BH

- In Schwarzschild coordinates. Surface's world line R(t) lies inside the light cone



- Light cone: $$0 = ds^2 = -(1-2M/R)dt^2 + \frac{dr^2}{1-2M/R}$$

- Cone's squeezing: $$\frac{dt}{dr} = \pm\frac{1}{1-2M/R}$$

- Peculiarity as $r \rightarrow 2M$: Squeezing down of light cones prevents surface's world line from reaching $2M$ as (coordinate time) $t \rightarrow \infty$
- The proper time of falling observer is finite

$$\tau \simeq \frac{\pi}{2}\left(\frac{R_o^3}{2M}\right)^{\frac{1}{2}} = 15\,\text{microseconds}\left(\frac{R_o}{2M}\right)^{3/2}\frac{M}{M_\odot} \quad \text{if } R_o \gg 2M$$

R_0 is initial radius of collapsing body

- Tidal forces finite at 2M

Eddington-Finkelstein coordinates

- A.Eddington (1922), D.Finkelstein (1958):

$$\boxed{\tilde{t} = t + 2M \ln |(r/2M) - 1|}$$

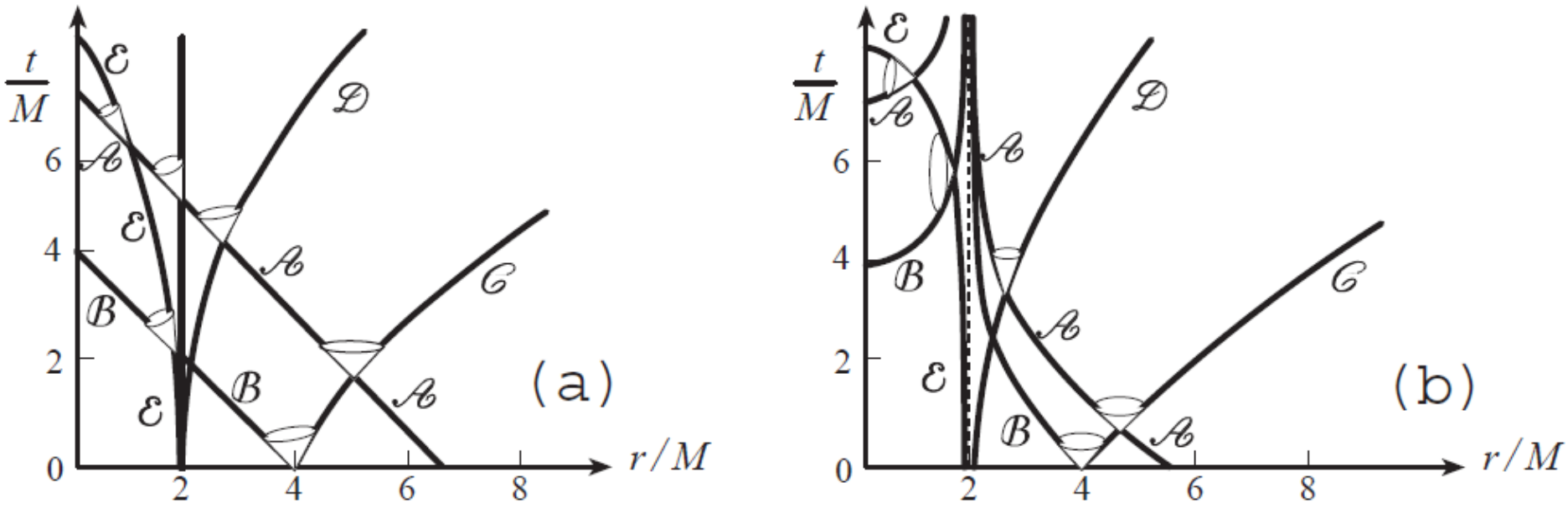
- No divergence at r=2M

$$\boxed{ds^2 = -\left(1 - \frac{2M}{r}\right) d\tilde{t}^2 + \frac{4M}{r} d\tilde{t}\, dr + \left(1 + \frac{2M}{r}\right) dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)}$$

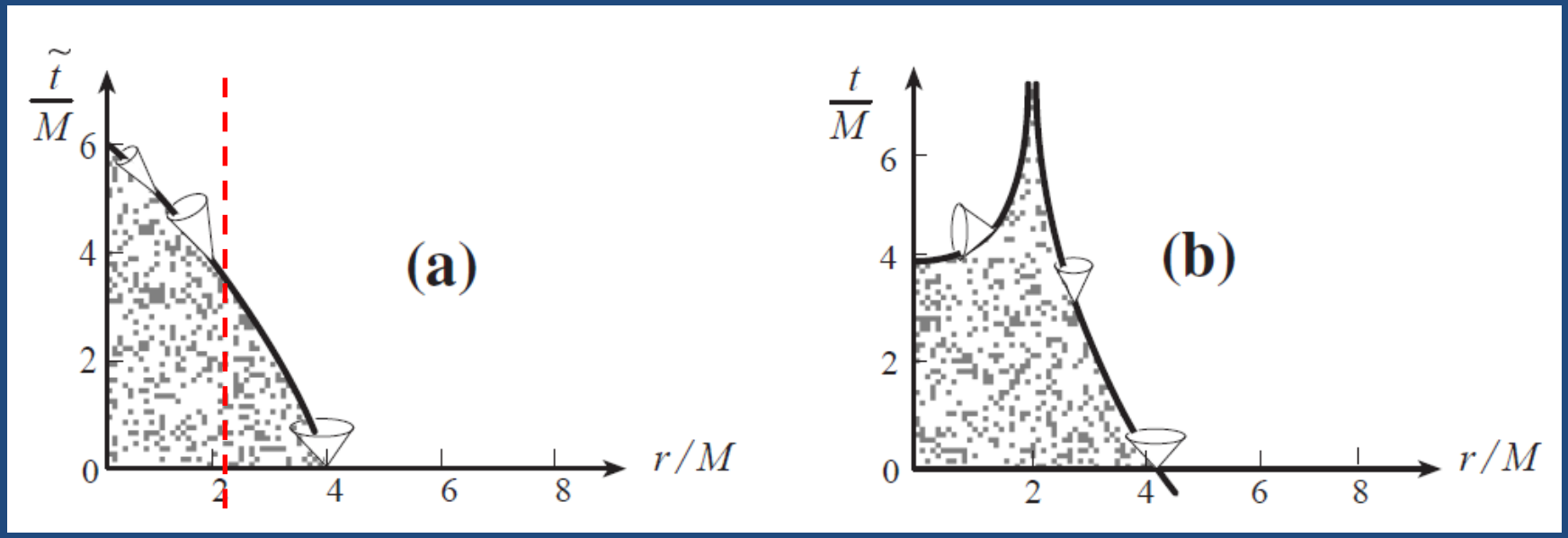
- No light cones' squeezing down

$$\frac{d\tilde{t}}{dr} = -1 \quad \text{for ingoing rays}$$

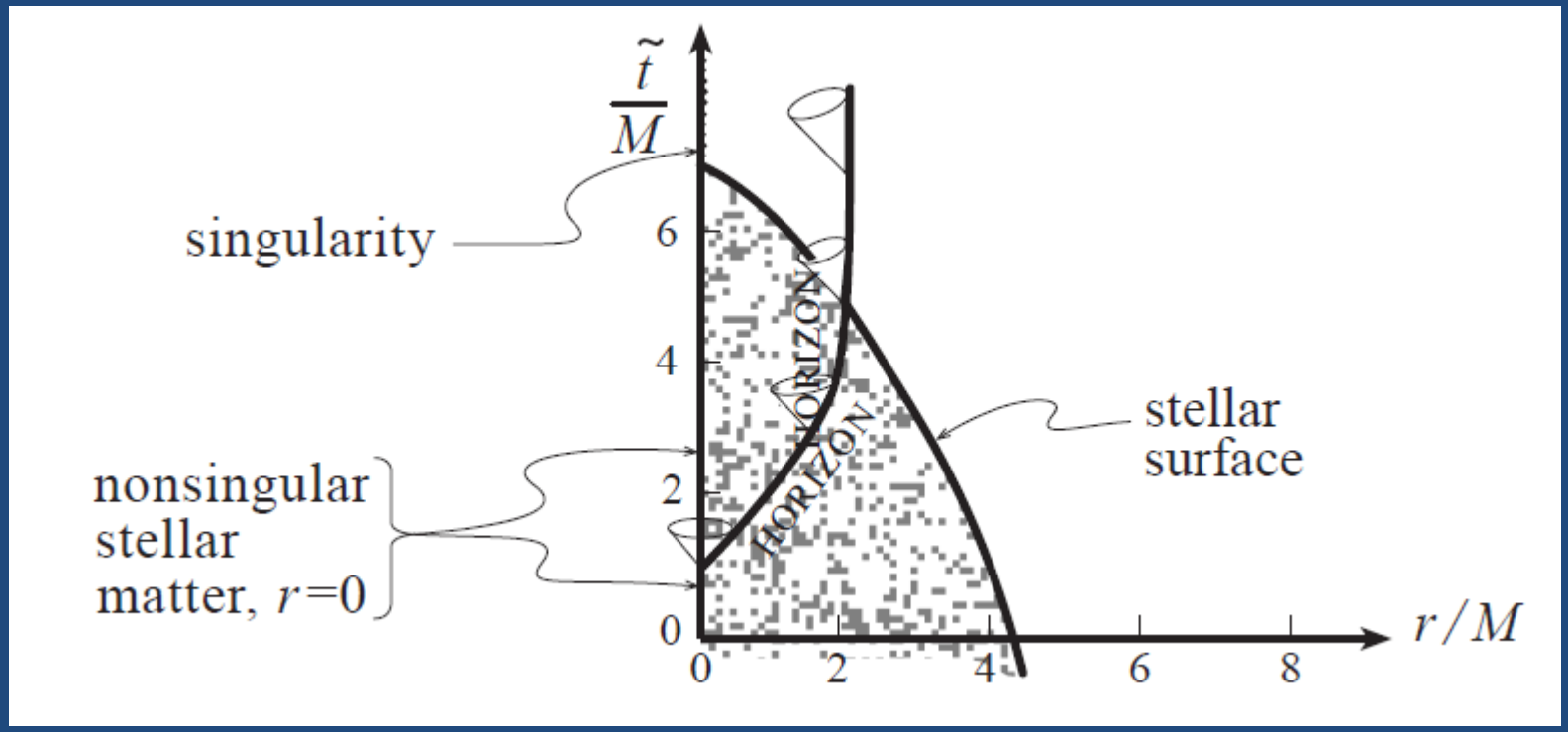
$$\frac{d\tilde{t}}{dr} = \left(\frac{1+2M/r}{1-2M/r}\right) \quad \text{for outgoing rays.}$$



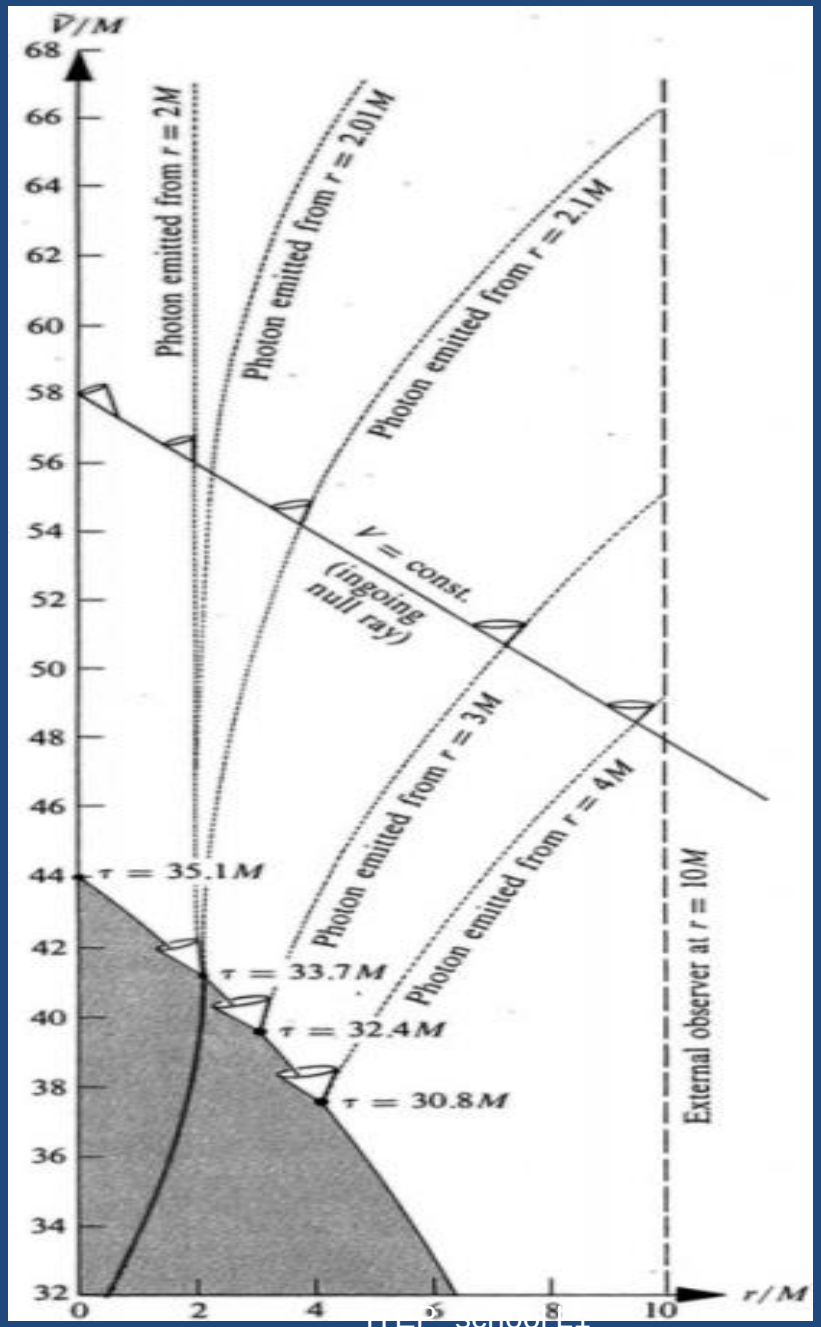
In E-F coordinates **In Schwarzschild coordinates**



Formation of a BH



- Event horizon hides central singularity of a BH
- **Cosmic censorship conjecture (R. Penrose): no naked singularities**


$\bar{v}/M$
68
66
64
62
60
58
56
54
52
50
48
46
44
42
40
38
36
34
32
0
2
4
6
8
10
r/M
Photon emitted from $r = 2M$
Photon emitted from $r = 2.01M$
Photon emitted from $r = 2.1M$
v = const.
(ingoing null ray)
Photon emitted from $r = 3M$
Photon emitted from $r = 4M$
$\tau = 35.1M$
$\tau = 33.7M$
$\tau = 32.4M$
$\tau = 30.8M$
External observer at $r = 10M$

Kruskal-Szekers coordinates

$$\left\{\begin{aligned} U &= \left(\frac{r}{2M}-1\right)^{1/2} e^{r/4M}\cosh\left(\frac{t}{4M}\right) \\ V &= \left(\frac{r}{2M}-1\right)^{1/2} e^{r/4M}\sinh\left(\frac{t}{4M}\right) \end{aligned}\right\} \quad r>2M$$

$$\left\{\begin{aligned} U &= \left(1-\frac{2M}{r}\right)^{1/2} e^{r/4M}\sinh\left(\frac{t}{4M}\right) \\ V &= \left(1-\frac{2M}{r}\right)^{1/2} e^{r/4M}\cosh\left(\frac{t}{4M}\right) \end{aligned}\right\} \quad r<2M$$

$$ds^2 = \frac{32M^3}{r} e^{-r/2M}(-dV^2+dU^2)+r^2(d\theta^2+\sin^2\theta d\varphi^2)$$

$$r=const: \quad U^2-V^2=const$$

$$r=2M: \quad V=\pm U$$

$$r=0: \quad V=+\sqrt{U^2+1}$$

$$t=const: \quad \begin{cases} \tanh\left(\frac{t}{4M}\right)=\frac{V}{U}, & r>2M \\ \tanh\left(\frac{t}{4M}\right)=\frac{U}{V}, & r<2M \end{cases}$$

$$ds^2=0: \quad V=\pm U+const$$

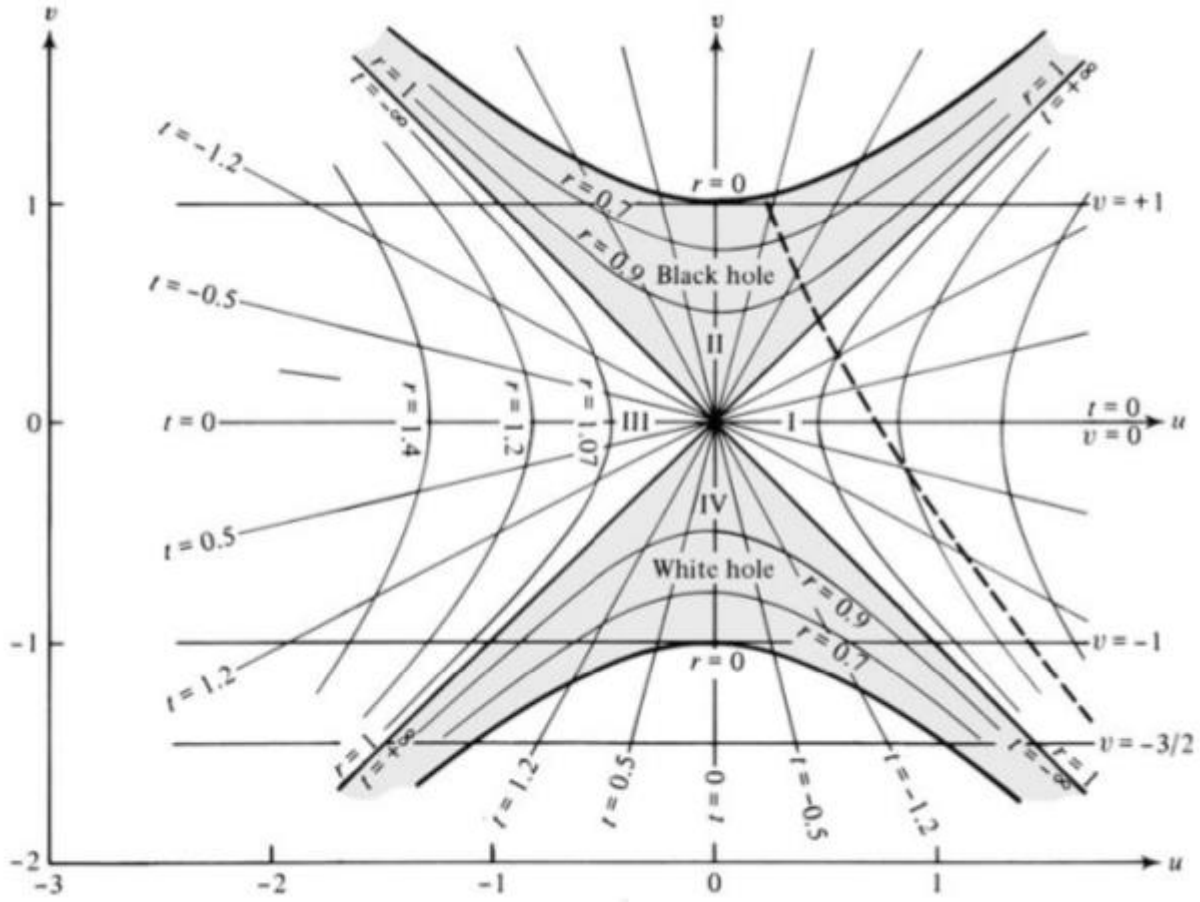
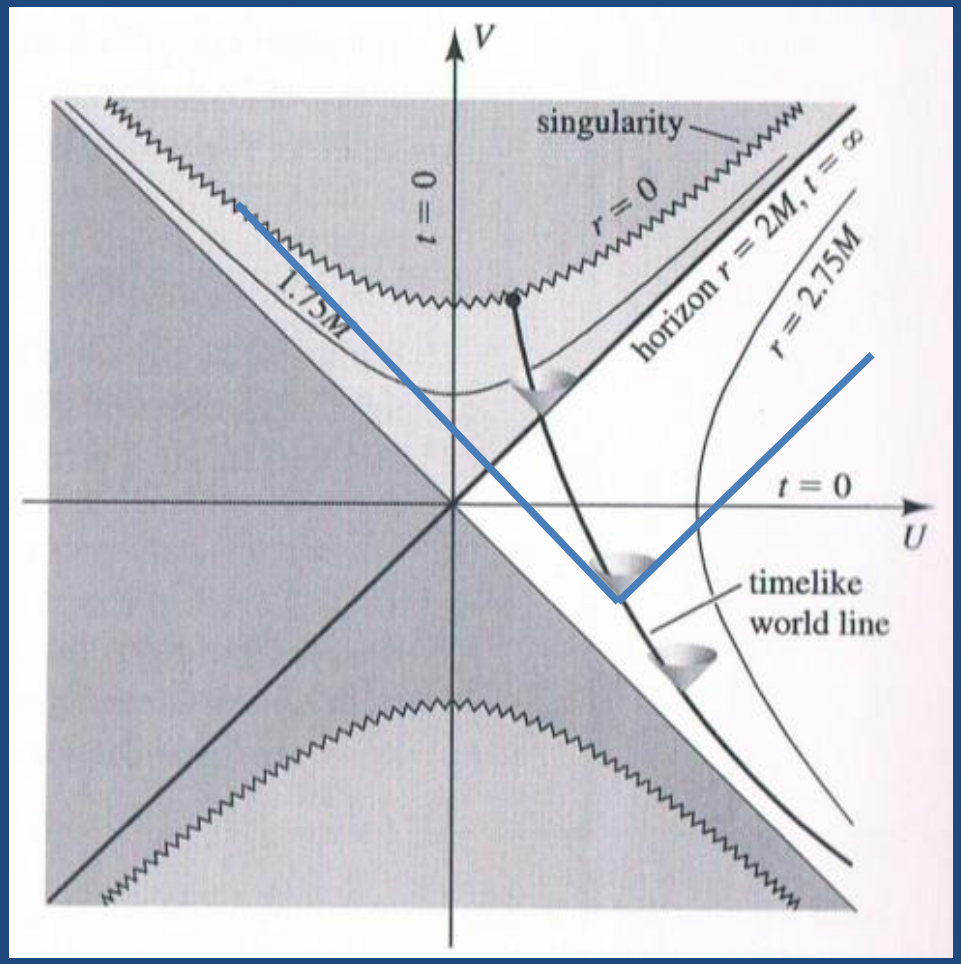
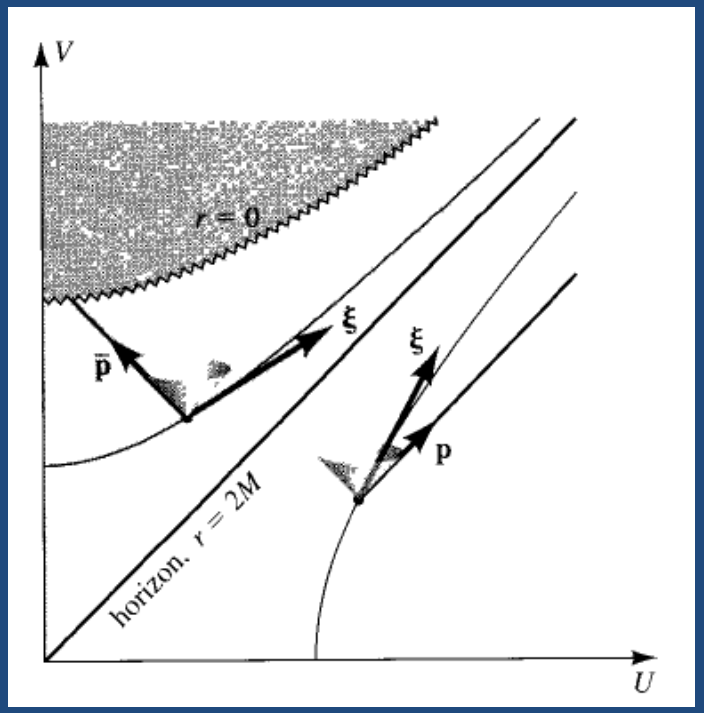


Fig. 8.6 The maximal Schwarzschild spacetime in Kruskal coordinates.

Falling to BH in K-S coordinates



Hawking evaporation



$$\xi\cdot\mathbf{p}+\xi\cdot\bar{\mathbf{p}}=0,\quad \xi=(1,0,0,0)\text{–}\mathit{Killing\ vector}$$

$$\xi\cdot\xi=g_{\alpha\beta}\xi^{\alpha}\xi^{\beta}=g_{tt}=-\left(1-\frac{2M}{r}\right)\ \Rightarrow$$

$$@\,r>2M:\quad \xi\quad \mathit{timelike},\qquad -\xi\cdot\mathbf{p}\propto-\mathbf{u}_{obs}\cdot\mathbf{p}=E_{obs}>0$$

$$\mathit{if}\quad \bar{\mathbf{p}}\ \mathit{were\ outside\ horizon}:\ -\xi\cdot\bar{\mathbf{p}}>0$$

$$\Rightarrow \mathit{violation\ of\ \ energy\ conservation}$$

$$@\,r<2M:\quad \xi\quad \mathit{spacelike},\ -\xi\cdot\bar{\mathbf{p}}<0$$

$$\mathit{it\ is\ not\ energy\ \ for\ any\ observer}$$

$$\mathit{it\ is\ comp.\ of\ \ spatial\ momentum}\ \left(\mathit{can\,be\ negative}\right)$$

Surface gravity of BH

- Near horizon

$$r = 2M,\ \theta = \theta_0, \varphi = \varphi_0$$
$$x = 2M \sin\theta_0 (\varphi - \varphi_0),\ y = 2M(\theta - \theta_0)$$
$$z = \int_{2M}^{r} dr / \sqrt{1 - 2M / r}$$

$$ds^2 = -(g_H\, z)^2 dt^2 + dx^2 + dy^2 + dz^2$$

$$g_H = \frac{1}{4M}$$

Rotating BH: Kerr spacetime

- Collapse of any non-rotating body → Schwarzschild BH (W. Israel)
- Collapse of any rotating body → Kerr BH (Hawking, Carter, Robinson +)

$$ds^2 = -\alpha^2 dt^2 + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \varpi^2 (d\phi - \omega dt)^2$$

$$\Delta = r^2 + a^2 - 2Mr\,, \quad \rho^2 = r^2 + a^2 \cos^2\theta\,, \quad \Sigma^2 = (r^2 + a^2)^2 - a^2 \Delta \sin^2\theta$$

$$\alpha^2 = \frac{\rho^2}{\Sigma^2}\Delta\,, \quad \varpi^2 = \frac{\Sigma^2}{\rho^2}\sin^2\theta\,, \quad \omega = \frac{2aMr}{\Sigma^2}$$

Boyer-Lindquist coordinates

- At r>>M

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 - \frac{4Ma}{r} \sin^2\theta d\phi dt + \left[1 + \mathcal{O}\left(\frac{M}{r}\right)\right] [dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$$

- Linearized gravity around rotating body (M,J):

$$\boxed{ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 - \frac{4J}{r} \sin^2\theta dt d\phi + \left(1 + \frac{2M}{r}\right) (dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2)}$$

- Parameter ***a*** has the sense of specific angular momentum (per unit mass) $Ma \equiv J_H$
- a→0 B-L coordinates → Schwarzschild coord.
- Stationary (independent of t) and axially symmetric (independent of ϕ)

Frame-dragging

- BH rotation drags inertially falling particles into circulatory orbital motion

$$\boxed{\Omega \equiv \frac{d\phi}{dt} = \frac{d\phi/d\tau}{dt/d\tau} = \frac{u^\phi}{u^t} = \omega}$$

$$\omega = \frac{2aMr}{\Sigma^2}$$

$$\Sigma^2 = (r^2 + a^2)^2 - a^2 \Delta \sin^2\theta$$

$$\Delta = r^2 + a^2 - 2Mr$$

- (aka Lense-Thirring precession)

Light cones and horizon

- $ds^2=0$, dr/dt maximal (light) if $d\phi = \omega dt$ (light cones rotate with LT frequency)

$$\frac{dr}{dt} = \pm\frac{\alpha\sqrt{\Delta}}{\rho} = \pm\frac{\Delta}{\Sigma}$$

$$\Delta = r^2 + a^2 - 2Mr$$

$$\Sigma^2 = (r^2 + a^2)^2 - a^2\Delta\sin^2\theta$$

- Light-cone pitch off as $\Delta \rightarrow 0$

$$r = r_H \equiv M + \sqrt{M^2 - a^2}$$

- As $r \rightarrow r_H$

$$r = r_H\ , \quad \theta = \text{constant}\ , \quad \phi = \Omega_H t + \text{constant}$$

$$\Omega_H = \omega(r = r_H) = \frac{a}{2Mr_H}$$

Horizon's rotation

Kerr ingoing coordinates

- Remove pathology of Boyer-Lindquist coordinates at $r=r_H$: Kerr coordinates (e.g.)

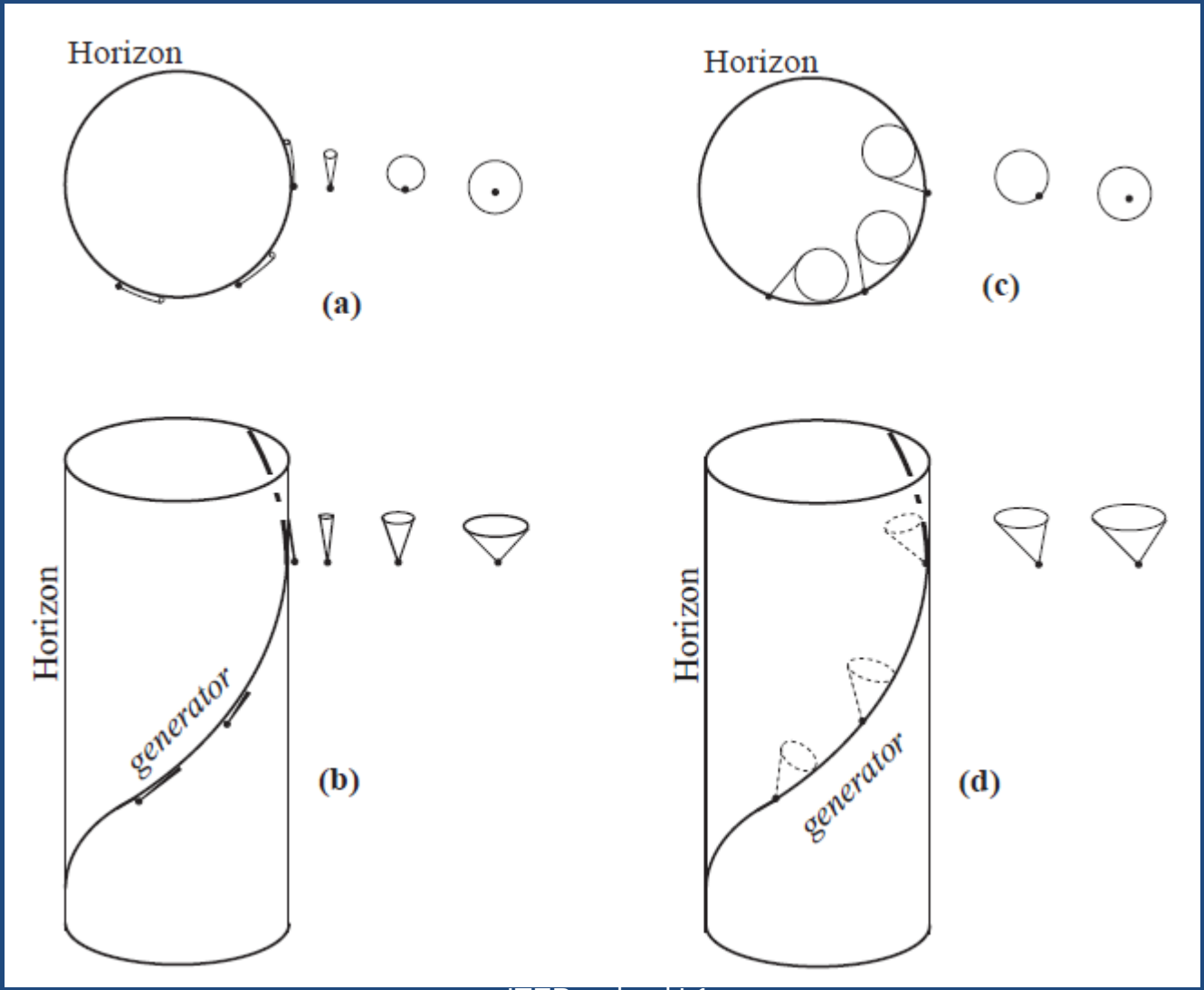
$$\tilde{t} = t + \int \frac{2Mr}{\Delta} dr \,, \quad \tilde{\phi} = \phi + \int \frac{a}{\Delta} dr$$

$$ds^2 = -\alpha^2 d\tilde{t}^2 + \frac{4Mr\rho^2}{\Sigma^2} dr d\tilde{t} + \frac{\rho^2(\rho^2+2Mr)}{\Sigma^2} dr^2 + \rho^2 d\theta^2 + \varpi^2 \left[d\tilde{\phi} - \omega d\tilde{t} - \frac{a(\rho^2+2Mr)}{\Sigma^2} dr \right]^2$$

- Transform to Eddington-Finkelstein coordinates as $a \to 0$

In B-L coordinates

In Kerr coordinates



ISCO

$$t: \quad -e = \boldsymbol{\xi}\cdot\mathbf{u} = g_{tt}u^t + g_{t\varphi}u^\varphi$$

$$\varphi: \quad l = \boldsymbol{\eta}\cdot\mathbf{u} = g_{\varphi t}u^t + g_{\varphi\varphi}u^\varphi$$

$$\mathbf{u}\cdot\mathbf{u} = g_{\alpha\beta}u^\alpha u^\beta = -1, \quad \sin\theta = 1$$

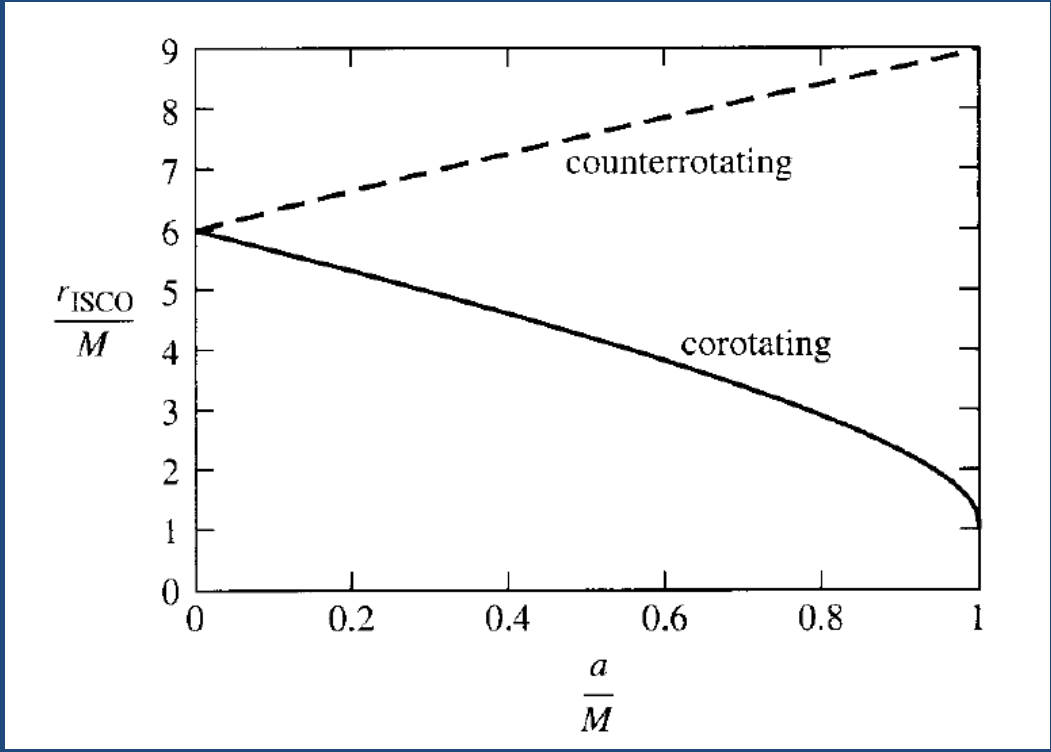
$$\underbrace{\frac{e^2-1}{2}}_{E} = \frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 + V_{eff}(r,e,l)$$

$$V_{eff}(r) \equiv -\frac{M}{r} + \frac{l^2 - a^2(e^2-1)}{2r^2} - \frac{M(l-ae)^2}{r^3}$$

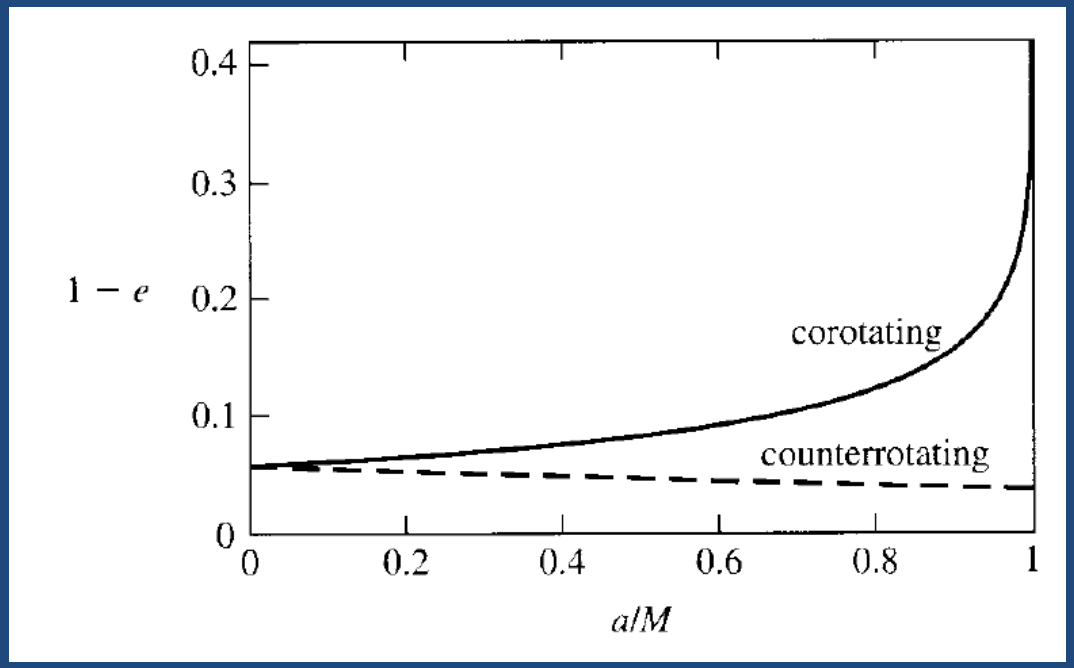
$$ISCO:$$

$$\frac{\partial V_{eff}(r)}{\partial r} = 0, \quad \frac{\partial^2 V_{eff}(r)}{\partial r^2} > 0$$

$$e = 1/\sqrt{3}, \quad l = 2M/\sqrt{3}, \quad R_{ISCO} = M$$



Binding energy at ISCO

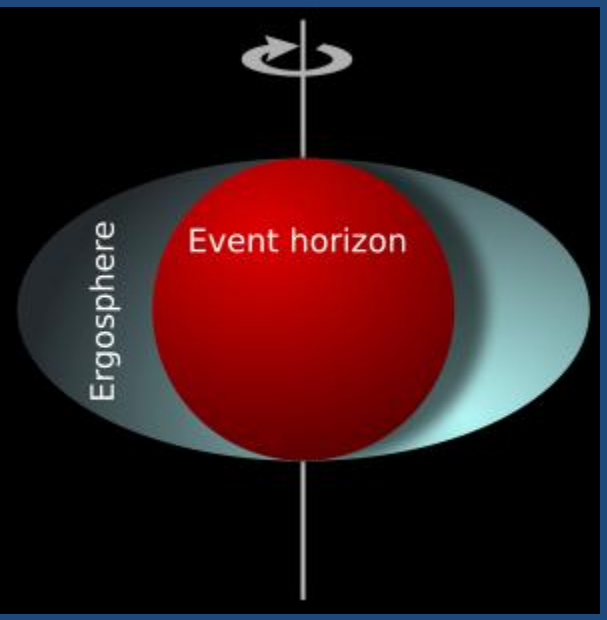


For a/M=1 1-e~ 0.42

Ergosphere

- $\partial/\partial t \cdot \partial/\partial t = g_{tt} = -\alpha^2 + \varpi^2\omega^2$ positive inside

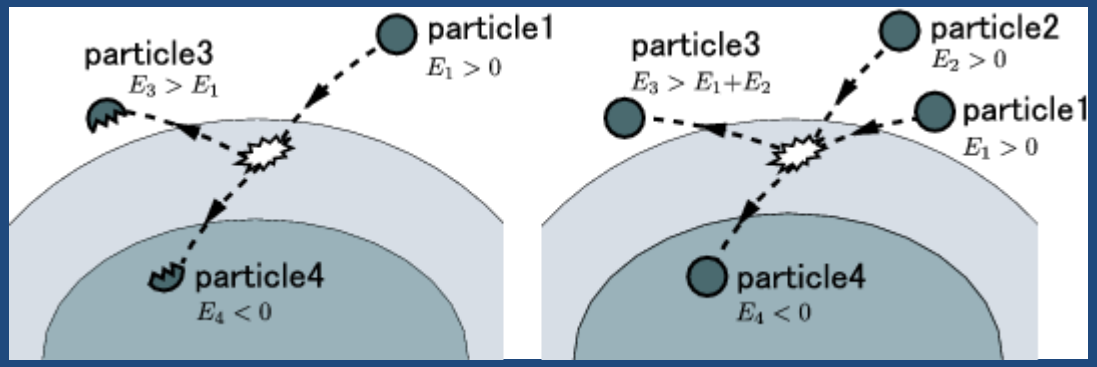
$$r < r_{\text{ergo}} \equiv M + \sqrt{M^2 - a^2\cos^2\theta}$$



- g_{tt} acts as a purely spatial metric component inside ergosphere
- Outside ergosphere particles can be static relative to distant observer
- Inside ergosphere particles should rotate to retain their time-like motion

Energy extraction from rotating BH

- R.Penrose (1969) thought experiment



$$\mathcal{E}_\infty^{\rm in} = -\vec{p}^{\rm in} \cdot \vec{U} = -\vec{p}^{\rm in} \cdot (\partial/\partial t) = -p_t^{\rm in}$$

$$\mathcal{E}_\infty^{\rm plunge} \equiv -p_t^{\rm plunge} \qquad \mathcal{E}_\infty^{\rm out} \equiv -p_t^{\rm out}$$

$$\vec{p}^{\rm in} = \vec{p}^{\rm plunge} + \vec{p}^{\rm out} \quad \Rightarrow \quad \mathcal{E}_\infty^{\rm out} = \mathcal{E}_\infty^{\rm in} - \mathcal{E}_\infty^{\rm plunge}$$

$$\mathcal{E}_\infty^{\rm plunge} = -\vec{p}^{\rm plunge} \cdot (\partial/\partial t)$$

Can be made negative in ergosphere! $\mathcal{E}_\infty^{\rm out} > \mathcal{E}_\infty^{\rm in}$

Irreducible mass

- $\mathcal{E}_\infty^{\rm out} > \mathcal{E}_\infty^{\rm in}$ hence $\Delta M = -(\mathcal{E}_\infty^{\rm out} - \mathcal{E}_\infty^{\rm in}) = \mathcal{E}_\infty^{\rm plunge} < 0$
- S. Hawking, 2d law of BH mechanics: ***horizon area never decreases, $dA_H \geq 0$***

$$A_H = 4\pi(r_H^2 + a^2) = 8\pi M r_H \quad \text{for a spinning hole}$$

$$A_H = 16\pi M^2 \quad \text{for a nonspinning hole, } a = 0 \; .$$

$$M_{\rm irr} = \sqrt{\frac{A_H}{16\pi}} = \sqrt{\frac{M(M + \sqrt{M^2 - a^2}\,)}{2}}$$

How much energy can be extracted?

$$M_{\rm rot} = M - M_{\rm irr} = M\left[1 - \sqrt{\frac{1}{2}\left(1 + \sqrt{1 - a^2/M^2}\right)}\right]$$

- For a=M: $M_{\rm rot} = M(1 - 1/\sqrt{2}) \simeq 0.2929M$
- First law of BH mechanics

$$\Delta M = \Omega_H \Delta J_H + \frac{g_H}{8\pi}\Delta A_H$$

$$r = r_H \equiv M + \sqrt{M^2 - a^2}$$

- BH surface gravity $g_H = \dfrac{r_H - M}{2Mr_H}$

g_H/α

Gravitational acceleration

BH thermodynamics

- Temperature of the horizon (S.Hawking, 1973)

$$\rho_a = C \exp\left[\frac{-\mathcal{E}_{a\,\infty} + \Omega_H \cdot \mathbf{j}_a}{k_B T_H}\right]$$

$$\boxed{T_H = \frac{\hbar}{2\pi k_B} g_H}$$

$$T_H \sim \frac{\hbar}{8\pi G M_H/c^3} \simeq \frac{6\times 10^{-8}\mathrm{K}}{M_H/M_\odot}\,, \quad \Omega_H \sim \frac{J_H}{M_H(2GM_H/c^2)^2}$$

- BH entropy (J.Bekenstein)

$$S_H = k_B \frac{A_H}{4{l_P}^2} \sim 1\times 10^{77} k_B \left(\frac{M}{M_\odot}\right)^2$$

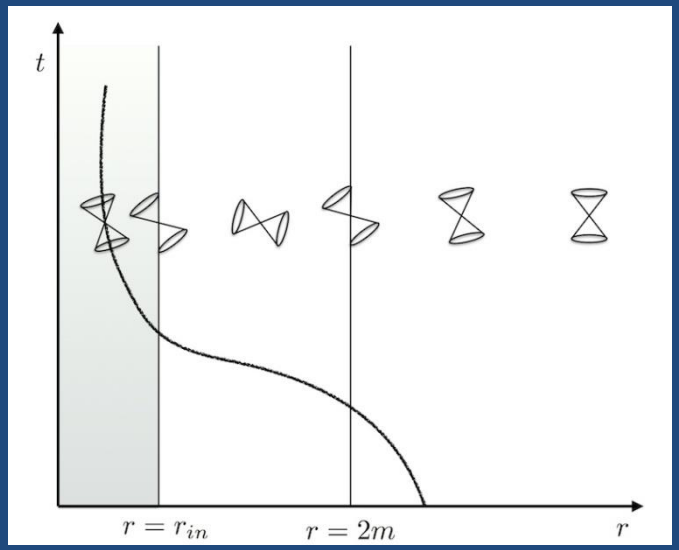
$$dM_H = T_H dS_H + \Omega_H \cdot d\mathbf{J}_H$$

BH evaporation

$$dE/dt = \sigma_B T_H^4 A_H \sim M^{-2}$$

$$t_{evap} = M/(dE/dt) \sim M^3$$

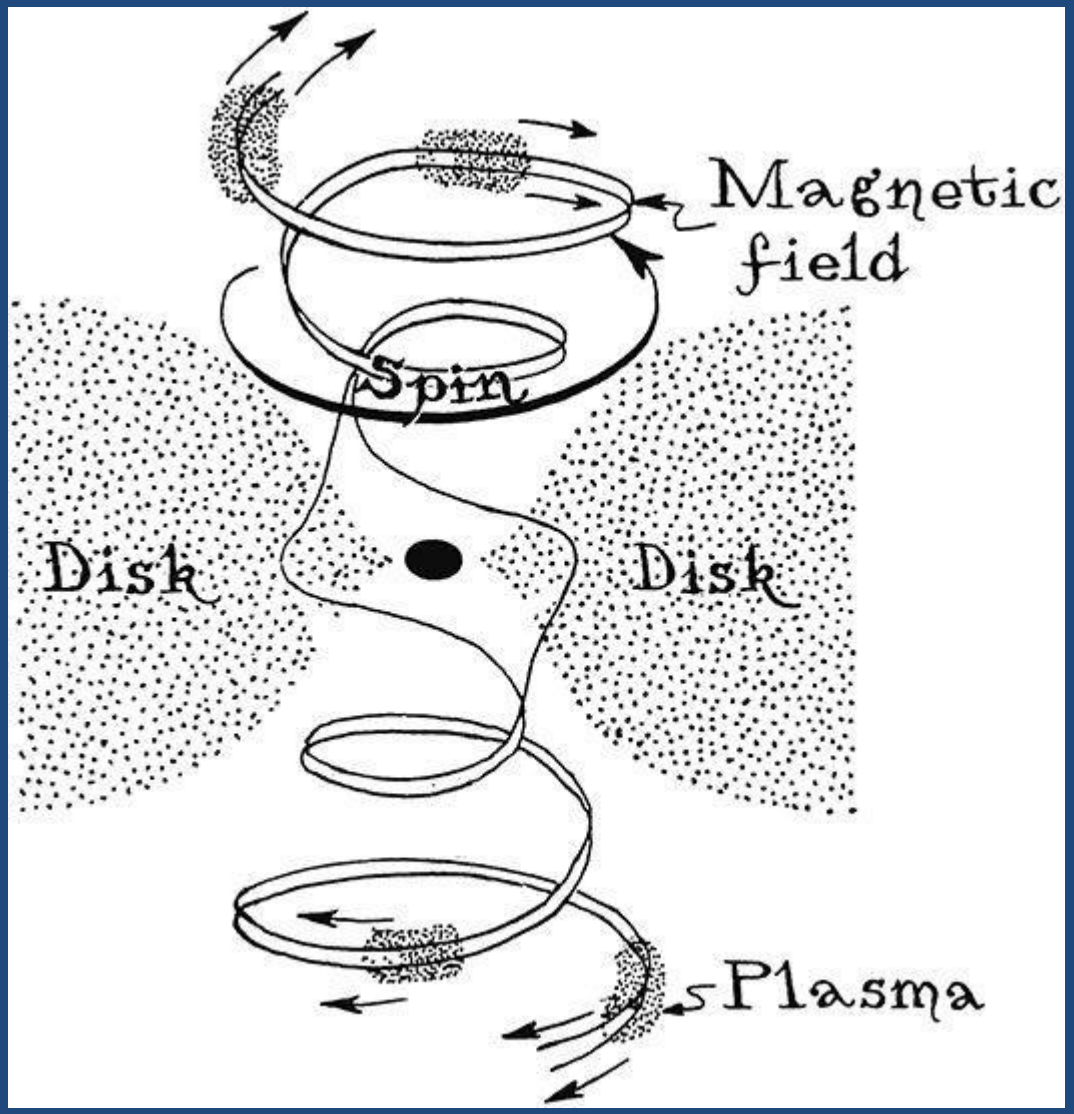
$$t_{evap} \sim 10^{10}\, yr (M/10^{15}\, g)^3$$



- Problems: informational paradox
- BH evaporation remnants (e.g. Planck stars, C.Rovelli, F.Vidotto arXiv:1401.6562

Blandford-Znajek mechanism

- Penrose process is difficult to realize in practice.
- If there is external magnetic field (e.g., accretion disk around BH), it can mediate the Penrose process
- Inside ergosphere, magnetic field lines must rotate → electric field appears near the horizon → possibility of pair creation (like in pulsars) → negative energy flux into BH, positive energy flux (Poynting) outwards
- Popular mechanism of jet formation in AGN and quasars, and maybe in GRB!


Magnetic field
Spin
Disk
Disk
Plasma