Quantization in background scalar fields

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Abstract

We consider $(0+1)$ and $(1+1)$ dimensional Yukawa theory in various scalar field backgrounds, which are solving classical equations of motion. The $(0+1)$–dimensional theory we solve exactly. In $(1+1)$–dimensions we consider background fields of the form $\phi_{cl} = Et$ and $\phi_{cl} = Ex$, which are inspired by the constant electric field. Here E is a constant. We study the backreaction problem by various methods, including the dynamics of a coherent state. We also calculate loop corrections to the correlation functions in the theory using the Schwinger–Keldysh diagrammatic technique. This poster is based on the paper [1].

1. Motivation

- In many cases, it is important to find the response of quantum system to a strong external field
- In particular, this problem arises when one try to describe the dynamics of fields on the background of collapsing matter (Hawking effect [2]), strong electric field (Schwinger effect [3]), expanding Universe [4], etc.
- Here we consider a simplified model to capture the general behavior of strong fields, which remains elusive in more complex systems
- As an example of such a **non-equilibrium situation** we propose to consider the Yukawa theory of interacting fermions and massless bosons in $(D+1)$-dimensional Minkowski spacetime (we start with $D = 0, 1$):

$$S = \int d^{D+1}x \left[\frac{1}{2}(\partial_\mu\phi)^2 + i\bar\psi\not{\partial}\psi - \lambda\phi\bar\psi\psi\right]. \tag{1}$$

- We assume that there is a strong scalar field, i.e. a classical solution $\phi_{cl}(x) \gg 1$ for some values of $(D+1)$-dimensional x; at the same time, $\psi_{cl} = 0$
- We split each field into the sum of the "classical background" and "quantum fluctuations": $\phi = \phi_{cl} + \phi_q$, $\psi = \psi_q$, quantize the "quantum" part, find tree-level correlation functions and evaluate loop corrections
- Our main goal is to find the **response of the scalar field** on such a background

2. Strong scalar field in 1D

First of all, let us consider 1D case, i.e. action (1) with $D = 0$ and Grassmanian ψ. The equations of motion in this case have have the following classical solution:

$$\phi_{cl}(t) = \frac{m}{\lambda} + Et, \qquad \psi_{cl} = \bar\psi_{cl} = 0, \tag{2}$$

which we will consider as a background. It can be shown that the tree–level expectation value of the equal-time product of two fermion operators does not depend on time:

$$\langle 0|\,\bar\psi\psi\,|0\rangle = 0 \quad \text{and} \quad \langle 1|\,\bar\psi\psi\,|1\rangle = 1, \tag{3}$$

where $\hat a|0\rangle = \hat a^+|1\rangle = 0$, $\hat a^+$ and $\hat a$ are fermion creation and annihilation operators. Furthermore, these identities are exact due to the commutation $[\bar\psi\psi, H_{full}] = 0$. Therefore, the backreaction problem for the scalar field is as follows:

$$\langle\ddot\phi\rangle \equiv -\lambda\langle 0|\bar\psi\psi|0\rangle = 0 \quad \text{or} \quad \langle\ddot\phi\rangle \equiv -\lambda\langle 1|\bar\psi\psi|1\rangle = -\lambda. \tag{4}$$

To complete the solution of the problem, let us calculate loop corrections to the scalar field propagator. We do this in two distinct ways. First, we directly calculate scalar two-point functions in the **operator formalism**, i.e. using mode decomposition for the fields and expanding the evolution operator. Second, we calculate the same quantities using **Schwinger–Keldysh diagrammatic technique**. We emphasize that the standard Feynman diagrammatic technique is not applicable in this case due to the time dependence of the Hamiltonian.

For the expectation value in the state $|0\rangle$ we obtain that the second order corrections are exactly zero. For the expectation value in the state $|1\rangle$ we obtain the following second-order correction for the scalar Keldysh propagator:

$$D^K(t_1,t_2) = D_0^K(t_1,t_2) + \Delta\langle\phi_1\rangle\,\Delta\langle\phi_2\rangle = \frac{1+t_1t_2}{2} + \frac{\lambda^2}{4}(t_1-t_0)^2(t_2-t_0)^2, \tag{5}$$

where t_0 is the moment when the interaction is adiabatically turned on. The corrections to the retarded/advanced propagators are bounded in time.

Let us emphasize several points:

- The calculations of two methods coincide (for the scalar ground state), as it was expected
- The expression (5) is nothing but the product of two disconnected corrections to the one-point functions
- Therefore, the growth of the quantum corrections does not imply the growth of the level population and anomalous quantum average
- Instead of this, eq. (5) means that corrections modify the scalar field mode decomposition:

$$\hat\phi(t) = \left(\frac{m}{\lambda} - \frac{\lambda t_0^2}{2}\right) + (E + \lambda t_0)\,t - \frac{\lambda}{2}t^2 + \frac{1}{\sqrt2}\left[(\hat\alpha + \hat\alpha^+) + i(\hat\alpha^+ - \hat\alpha)\,t\right] \tag{6}$$

- It can be proven that the identity (5) is exact, i.e. higher-loop corrections do not modify it

3. Linearly growing in time scalar field in 2D

Now let us consider 2D case, i.e. action (1) with $D = 1$ and Dirac fermions. We choose the following background:

$$\phi_{cl} = \frac{m}{\lambda} + Et, \qquad \psi_{cl} = \bar\psi_{cl} = 0. \tag{7}$$

We decompose the fermion field in positive- and negative-frequency modes, which solve the corresponding equation of motion, obey the equal-time anti-commutation relations and tend to plane waves at the past and future infinities. This way we explicitly find the response of Dirac field. The positive-frequency modes look as follows:

$$\psi^{(+)}(t,x) = e^{-\frac{\pi p^2}{4\lambda E}} \begin{pmatrix} D_\nu(z(t)) \\ \frac{1+i}{\sqrt2}\frac{p}{\sqrt{2\lambda E}} D_{\nu-1}(z(t)) \end{pmatrix} e^{ipx}, \tag{8}$$

where $\nu = -\frac{ip^2}{2\lambda E}$, $z(t) = \frac{1+i}{\sqrt{\lambda E}}\lambda E t$. The negative-frequency modes are obtained by the charge conjugation of this expression. Note that technically parabolic cylinder functions **do not tend** to plane waves at the infinity, although they behave properly at large momenta ($|p| \gg \lambda E|t|$). Hence, positive- and negative-frequency modes, which are defined in this way, posses **correct UV behavior**. Thus we choose to consider such modes.

Quantizing the Hamiltonian of the theory and using Hamilton's equations, one obtains the following operator equation for the scalar field:

$$\partial^2\hat\phi + \lambda\hat{\bar\psi}\hat\psi = 0. \tag{9}$$

Hence, one needs to calculate the classical current $j_{cl}(t) = \langle\hat{\bar\psi}\hat\psi\rangle$ to find the response of the classical field $\phi_{cl} = \langle\hat\phi\rangle$. Using modes (8), we obtain the following tree-level current:

$$\langle\bar\psi\psi\rangle(t) \approx \frac{\lambda\phi_{cl}}{\pi}\log\frac{\lambda\phi_{cl}}{\Lambda}. \tag{10}$$

Note that in the limit $\lambda \to 0$ this expression correctly reproduces the current in the free theory. This points us to the fact that despite the non-stationarity of the theory under consideration it is **well approximated by the free theory with time-dependent mass**.

In order to check this we calculate one-loop corrections to the vertex, scalar and fermion propagators using Schwinger–Keldysh diagrammatic technique. We confirm that these **corrections do not grow in time** and vanish in the limit $\lambda \to 0$. Hence, in the limit of small coupling constant time–dependent corrections to the tree-level correlation functions (including scalar current) are negligible despite the strength of the background. This type of behavior does not resemble the one in strong electric [3] or gravitational [2, 4] fields, in which loop corrections to these quantities do grow with time.

4. Linearly growing in time scalar field in 2D

We also consider another background in the $D = 1$ theory:

$$\phi_{cl} = \frac{m}{\lambda} + Ex, \qquad \psi_{cl} = \bar\psi_{cl} = 0. \tag{11}$$

In this case one should be more careful with the mode decomposition, but similarly to the case $\psi_{cl} = \frac{m}{\lambda} + Et$ one can define "positive" and "negative" frequency fermion modes and construct them from the corresponding parabolic cylinder functions. Using these modes we again calculate the tree-level current and **reproduce** (10) **in the leading order**. However, we emphasize that the subleading order corrections to (10) are different in the $\psi_{cl} = \frac{m}{\lambda} + Et$ and $\psi_{cl} = \frac{m}{\lambda} + Ex$ cases.

Similarly to the $\psi_{cl} = \frac{m}{\lambda} + Et$ we calculate loop corrections in the Schwinger–Keldysh diagrammatic technique and find that they **also do not grow with time**.

5. Coherent state decay

Finally, we consider the decay of the coherent state $|\phi_{cl}\rangle$, i.e. such state that

$$\langle\phi_{cl}|\hat\phi(t=0,x)|\phi_{cl}\rangle = \phi_{cl}(x) \qquad \text{with} \qquad \phi_{cl} = \frac{m}{\lambda} + Ex. \tag{12}$$

In other words, we consider a self-guided dynamics of the once created field ϕ_{cl}. To the best of our knowledge, this approach has not yet been considered for other non-equilibrium systems.

However, we have found that the behavior of the correlation functions in this case is **qualitatively the same** as the one previously found for the strong fixed scalar backgrounds.

6. Discussion

As soon as the dynamics in the strong scalar field (when ϕ'_{cl} is small, while ϕ_{cl} itself is large) is weakly sensitive to the choice of the ground state, we can estimate its **effective action** in the equilibrium approach. It is a standard textbook exercise to show that in this approach the effective action for the scalar field (the action one obtains after the integration over the fermion degrees of freedom) in the leading order looks as follows:

$$S_{eff} = \int d^2x\left[\frac{1}{2}(\partial_\mu\phi)^2 - V_{eff}[\phi]\right], \quad \text{where} \quad V_{eff}[\phi] \approx \frac{(\lambda\phi)^2}{2\pi}\log\frac{\phi}{\langle\phi\rangle_{GS}} - \frac{(\lambda\phi)^2}{4\pi} \tag{13}$$

and $\langle\phi\rangle_{GS}$ is the minimum of the renormalized effective potential $V_{eff}[\phi]$. Thus, the calculation with the use of the Feynman approach shows that zero point fluctuations of the fermion field polarize the vacuum and deform the classical scalar field background. However, we remind that this calculation is valid only if $|\not{\partial}\phi| \ll \lambda\phi^2$ and $\lambda \to 0$ (in the opposite case loop corrections to the level population and anomalous quantum average are non-zero). Both of these conditions hold in the limit $\lambda \to 0$, $t \to \infty$ for $\phi_{cl} = Et$ or $\lambda \to 0$, $x \to \infty$ for $\phi_{cl} = \frac{m}{\lambda} + Ex$. Obviously, they also hold near the minimum of the effective potential. Therefore, in this limit the scalar field just **classically rolls down to the minimum of such a potential**.

References

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