Superfield realization of hidden R-symmetry in extended SYM theories and its applications

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Introduction

- Supersymmetry imposes strong restrictions on the structure of effective action
- It is convenient to use superspace to study effective action. In theories with extended supersymmetry, the concept of harmonic superspace plays an important role.
- The standard approach of studying the low-energy effective action for $\mathbf{4}D$, $\mathcal{N}=\mathbf{4}$ and $\mathbf{5}D$, $\mathcal{N}=\mathbf{2}$ SYM theories uses supersymmetry transformations
- An alternative approach was found in this paper using R symmetry instead of supersymmetry

4D harmonic superspace

Standard superspace

$$z^M = (x^m, \theta^\alpha_i, \bar\theta^{\dot\alpha i})$$

Central basis

$$Z = (z, u) = (x^m, \theta^\alpha_i, \bar\theta^{\dot\alpha i}, u^{\pm i})$$

$u^{\pm i}$ — harmonics parameterizing the sphere $SU(2)_R/U(1)$ ($u^{+i}u^-_i = 1$, $\overline{u^{+i}} = u^-_i$)

Analytical basis

$$Z_{(an)} = (x^m_{(an)}, \theta^\pm_\alpha, \bar\theta^\pm_{\dot\alpha}, u^{\pm i})$$

$$x^m_{an} = x^m - 2i\theta^{(i}\sigma^m\bar\theta^{j)}u^+_i u^-_j \quad \theta^\pm_\alpha = u^\pm_i\theta^i_\alpha \qquad \bar\theta^\pm_{\dot\alpha} = u^\pm_i\bar\theta^i_{\dot\alpha}$$

$4D, \mathcal{N}=4$ SYM theory in harmonic superspace

- **$4D, \mathcal{N}=2$ vector multiplet**

  Classical action

$$S^{\mathcal{N}=2}_{\text{SYM}} = \frac{1}{2}\sum_{n=2}^{\infty}\text{tr}\frac{(-i)^n}{n}\int d^{12}z du_1\dots du_n \frac{V^{++}(z,u_1)\dots V^{++}(z,u_n)}{(u^+_1u^+_2)\dots(u^+_nu^+_1)}$$

  Superfield strength

$$W = -\frac{1}{4}(\bar\nabla^+)^2V^{--} \qquad \bar W = -\frac{1}{4}(\nabla^+)^2V^{--}$$

  Zero curvature conditions

$$D^{--}V^{++} - D^{++}V^{--} + i[V^{++}, V^{--}] = 0$$

- **q - hypermultiplet**

  Analytical superfield

$$q^+ = q^+(x^m, \theta^+_\alpha, \bar\theta^+_{\dot\alpha}, u^{\pm i})$$

  Doublet q^+_a

$$q^+_a = (q^+, -\widetilde{q}^+) \qquad \widetilde{q^+_a} = q^{+a} = \begin{pmatrix}\widetilde{q}^+\\ q^+\end{pmatrix}$$

  Action

$$S_q = \frac{1}{2}\int d\zeta^{-4}q^+_a\left(D^{++} + iV^{++}\right)q^{+a}$$

- **$4D$, $\mathcal{N}=4$ SYM action**

$$S^{\mathcal{N}=4}_{\text{SYM}} = S^{\mathcal{N}=2}_{\text{SYM}} + S_q$$

  Hypermultiplet q is in the adjoined representation of the gauge group

On $5D, \mathcal{N}=2$ SYM theory

- The $\mathbf{5}D$ harmonic superspace differs from $\mathbf{4}D$ only by spinor structure of Grassmann coordinates θ_α
- There is only one superfield strength

$$W = \frac{i}{8}(\nabla^+)^2V^{--}$$

  due to the other spinor structure
- The action of $\mathbf{5}D, \mathcal{N}=\mathbf{2}$ SYM theory has similar form as the action of $\mathbf{4}D, \mathcal{N}=\mathbf{4}$ one and possess similar symmetries

Symmetries of $4D$, $\mathcal{N}=4$ SYM action

- Manifest supersymmetry transformations
- Hidden supersymmetry transformations

$$\delta V^{++} = \left[\epsilon^{a\alpha}\theta^+_\alpha - \bar\epsilon^a_{\dot\alpha}\bar\theta^{+\dot\alpha}\right]q^+_a$$

$$\delta q^+_a = -\frac{1}{32}(D^+)^2(\bar D^+)^2\left[\epsilon^\alpha_a\theta^-_\alpha V^{--} + \bar\epsilon_{a\dot\alpha}\bar\theta^{-\dot\alpha}V^{--}\right]$$

- Manifest $SU(2)\times SU(2)$ R-symmetry transformations

$$\delta V^{++} = A^i_j\left(u^+_i\frac{\partial}{\partial u^{+j}} + u^-_i\frac{\partial}{\partial u^{-j}}\right)V^{++}$$

$$\delta q^{+a} = A^i_j\left(u^+_i\frac{\partial}{\partial u^{+j}} + u^-_i\frac{\partial}{\partial u^{-j}}\right)q^{+a}$$

$$\delta_{\text{PG}}q^{+a} = B^a_b q^{+b}$$

- Hidden R-symmetry transformations extending R-symmetry group to $SU(4)$

$$\delta V^{++} = \left[\lambda^{-a}(\theta^+)^2 + \bar\lambda^{-a}(\bar\theta^+)^2\right]q^+_a$$

$$\delta q^+_a = \frac{1}{64}(D^+)^2(\bar D^+)^2\Big[\lambda^+_a(\theta^-)^2V^{--} - 2\lambda^-_a\theta^{+\alpha}\theta^-_\alpha V^{--} + \bar\lambda^+_a(\bar\theta^-)^2V^{--} - 2\bar\lambda^-_a\bar\theta^+_{\dot\alpha}\bar\theta^{-\dot\alpha}V^{--}\Big]$$

$$\lambda^{-a} = \lambda^{ia}u^-_i,\ \bar\lambda^{-a} = \bar\lambda^{ia}u^-_i,\ \overline{\lambda_{ia}} = \bar\lambda^{ia},\ \lambda^{ia}$$

Deriving of the low-energy effective action

Leading term in the effective action

$$c\int d^{12}z du\ \ln\left(\frac{W}{\Lambda}\right)\ln\left(\frac{\bar W}{\Lambda}\right)$$

Hidden supersymmetry transformations → Effective action ← Hidden R-symmetry transformations

$$S = c\int d^{12}z\left[\ln\left(\frac{W}{\Lambda}\right)\ln\left(\frac{\bar W}{\Lambda}\right) + \mathcal{L}\left(-\frac{q^{ia}q_{ia}}{W\bar W}\right)\right]$$

$$\mathcal{L}(Z) = \sum_{n=1}^{\infty}\frac{Z^n}{n^2(n+1)} = (Z-1)\frac{\ln(1-Z)}{Z} + \text{Li}_2(Z) - 1$$

Λ — arbitrary scale

Conclusion

- Expressions for hidden transformations of R -symmetry in the harmonic superspace $\mathbf{4}D$, $\mathcal{N}=\mathbf{4}$ and $\mathbf{5}D$, $\mathcal{N}=\mathbf{2}$ SYM theory are obtained
- These transformations are applied to obtain the low-energy effective action of these theories
- It would be tempting to reveal other possible implications of hidden R-symmetry in extended superfield gauge theories in diverse dimensions, for example, $\mathbf{6}D$, $\mathcal{N}=\mathbf{(2,0)}$ SYM theory

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