Chapter 1

Behind and Beyond the Standard Model

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This is an uncompleted transcript of the lectures given at TASI 2015.

1. Introduction

During about the last ninety years, progress in the understanding of fundamental physics has largely coincided with progress in Quantum Field Theory (QFT). The formulation and the experimental verification of the Standard Model (SM) is the remarkable coronation of these nine decades of progress. But what is perhaps more remarkable and more peculiar to this quadrant of science is that, along the great empirical success of the SM, there lie great mysteries, paradoxes and tantalizing hints. To quote the most obvious ones

- Why is spacetime so flat and the observable universe so large?
- Why is gravity so weak compared to the other fundamental interactions?
- What is Dark Matter (DM) made of?
- Is there a rationale behind the structure of the SM? In particular, is there a unifying principle for fundamental forces?

It is quite likely that a full answer to all these questions lies beyond the realm of QFT. For instance it may require a fomulation where spacetime itself is emergent, as hinted by string theory in all its ramifications. But, as indicated in particular by our understanding of DM, the hierarchy problem and possiby unification, ample space for progress plausibly exists within QFT. Realizing that progress is the goal of the research area denominated

Physics Beyond the Standard Model or more simply (BSM).

As I believe there is no much value in describing just BSM models per se, I have structured these lectures into two parts. The first, which could be called *Behind the SM*, essentially describes the ideology underlying our understanding of particle physics. In particular a critical overview of the SM is given, insisting on its paradoxical success. The second part is the proper *Beyond the SM* lecture, describing two specific scenarios, Supersymmetry and Compositeness. This is therefore the outline

(I) Behind the SM
- Effective Field Theory (EFT) Ideology & Naturalness
- The SM as an EFT and the Hierarchy Paradox
- The Magic of the SM

(II) Beyond the SM
- Supersymmetry
- Compositeness

2. Behind the Standard Model

2.1. *Effective Field Theory Ideology*

According to our modern perspective, any description of a range of phenomena employing Quantum Field Theory (QFT) should be regarded as an *effective* one, valid only below some physical momentum cut-off Λ. The standard picture is that there appear degrees of freedom with a mass gap $\Lambda_{IR} \ll \Lambda$ and Effective Quantum Field Theory (EQFT) is employed to describe their interactions at energies $E \ll \Lambda$. In this situation, essentially by construction, the dynamics does not vary appreciably as E is varied between Λ_{IR} and Λ, and the system appears approximately scale invariant in this energy range. In this way any given effective QFT can thus be associated to a small perturbation of a specific fixed point of the renormalization group (RG). More technically we can write

$$\mathcal{L} = \mathcal{L}_{FP} + \mathcal{L}_{pert} \tag{1}$$

EQFTs are thus largely characterized by the possible fixed points of the RG, and by their neighbourhood, which describes their possible deformations.

In the simplest and most studied cases, the fixed point corresponds to a massless free field theory, while the perturbation is dominated by weak renormalizable interactions interactions and mass terms. In general, given

$\mathcal{L}_{FP}$ is scale invariant, the operators appearing in $\mathcal{L}_{pert}$ can be classified according to their scaling dimension, corresponding to the eigenvalue Δ under the action of the conserved dilation charge D. Depending on whether $\Delta < 4$, $\Delta = 4$ or $\Delta > 4$, the perturbations are termed *relevant*, *marginal* or *irrelevant*, corresponding to their behaviour as the the RG flows towards the infrared (IR). Indeed the coefficient g_Δ of an operator of dimension Δ has dimension $4 - \Delta$. By simple dimensional analysis its effects are given, within perturbation theory, by a power series in $g_\Delta E^{\Delta-4}$. Accordingly the most important effects at energies $E \ll \Lambda$ are expected from the relevant or marginal perturbations while the effects of irrelevant perturbations are suppressed by positive powers of E/Λ. As the relevant and marginal deformations are expectedly a few, while the irrelevant ones are an infinite set carrying in principle a great deal (possible all) of information about the microphysics, this property has crucial structural consequences, which we shall discuss in a moment.

2.1.1. *Fixed points, CFTs and RG flows*

An additional crucial aspect of EQFTs, as characterized by Eq. (1), is that in all known cases, the dilation invariance of $\mathcal{L}_{FP}$ is boosted to invariance under the full conformal group, which is isomorphic to $SO(4,2)$ in 4D. More specifically the upgrade of dilation invariance to conformal invariance appears valid for unitary field theories, that is field theories admitting a consistent quantum field theory intepretation. Indeed, by relaxing the request of unitarity one can construct field theories violating the rule. Such field theories may still be interesting to describe critical points of continuous classical systems at thermal equilibrium.

In QFT the implication scale $\rightarrow$ conformal is known since long to hold true for free field theories, and examples at finite coupling have been accumulating through the decades, in particular at large N and/or within supersymmetry. Moreover in the case of 2D QFT the implication was rigorously proven almost three decades ago by following the same logic underlying the proof of the c-theorem. Instead, in the 4D case for quite some time the difficulties in making progress paralleled the difficulties in getting a hold of the 4D analogue of the a-theorem. The proof of the the a-theorem and the methodology associate to it has however offered new insight into the connection scale $\rightarrow$ conformal in 4D. It was thus realized that, if it existed, an SFT, that is a theory that is scale invariant but not conformally invariant, should satisfy an infinite set of highly non trivial *vanishing identities*. These involve matrix elements between the vacuum and physical states of expressions involving an arbitrary number of powers of the trace of the energy momentum tensor T^μ_μ, starting with the second power. It was argued in that the only way for this to happen is if $T^\mu_\mu = 0$ which is equivaent to full conformal invariance. While this argument seems completely reasonable on phsyical grounds, a full fledged and rigorous proof seems still missing. What is certain is that in SFTs, if they exist, should be rather peculiar systems.

For the above reasons and because of the lack of explicit SFTs, scale invariance is normally taken to coincide with the conformal invariance. According to our line of reasoning any EQFT is therefore described by the a small deformation of a CFT. Hence the importance of CFT in our description of particle physics and also for depicting what may lie beyond the SM. This importance goes along with the fact that there is a lot that is known and and a lot that is being learned about CFT, because of the combined constraints set by symmetry and unitarity. For instance, a variant of the a-theorem strongly constrains the structure of RG flows in the neighbourhood of a CFT, that is to say for a CFT deformed by couplings that remain marginal and small throughout the RG flow. In particular such flows cannot asymptote towards closed cycles or more exotic behaviours such as strange attractors. Mainly because of unitarity the only allowed asymptotics must indeed satisfy $T^\mu_\mu = 0$ and are themselves conformal field theories. In particular SFT asymptotics are excluded. Stated in another way: if a small deformation of a CFT is itself scale invariant then it must be fully conformally invariant. Given massless free field theory in 4D i alwy conformally invariant this result also implies no perturbative SFT can exist in 4D On the other hand we have example of small deformations free field theory giving rise to new CFTs, these are the so called Banks-Zaks fixed points.

2.1.2. *Emergent Simplicity*

The perhaps most important physical consequence of Eq. (1) is the one we mentioned above: as the number of relevant or marginal deformations of a CFT is expected to be finite the apparent complexity of L dramatically decreases as E is lowered below the UV cut-off Λ. Provided Λ_{IR}/Λ is sufficiently small compared to the experimental precision, at the scale of this ower threshold, observables are effectively insensitive to the virtually infinite amount of structure stored in the infinite tower of irrelevant coupling, and physics is adequately described by the finitely many coefficients of operators with dimension $\Delta \leq 4$. In the case where the fixed point is free, these precisely coincide with the set of renormalizable couplings, which essentially define the canonical, older, perspective on QFT. In that perspective renormalizability is a working assumption realizing the request of predictivity. In the modern perspective, renormalizability is instead viewed as the unavoidable IR fate of EQFTs possessing a broad separation of scales, $\Lambda_{IR} \ll \Lambda$.

One corollary of the IR loss of complexity along the RG flow is the emergence of accidental (approximate) symmetries. Indeed, quite often, given the gauge group and the field content, the scalar operators with $\Delta \leq 4$ happen to be singlets of some global symmetry group $\mathcal{H}$. The simplest occurrence of this phenomenon is illustrated by QED, whose field content is just given by the photon field A_μ and by the 4-spinor electron field ψ. Outlining the $\Delta \leq 4$ operators the most general lagrangian can be written as

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \bar{\psi}i\not{D}\psi - m_e\bar{\psi}\psi + \mathcal{L}_{\Delta>4} \tag{2}$$

IR physics is dominated by just three terms. It turns out each of these terms is even under the action of parity P. On the other hand, already at dimension 5 and 6 we can find P-off scalar operators. In parrticular at $\Delta = 5$ we have $\bar{\psi}\gamma_5\sigma_{\mu\nu}\psi F^{mu\nu}$ corresponding to an electric dipole moment for the electron, while at $\Delta = 6$ we have for instance $\psi\bar{\gamma}^\mu\psi\psi\bar{\gamma}_5\gamma_\mu\psi$. Partity is a very inportant feature of low energy electromagnetic interactions, QED in particular, however ffrom the modern EQFT perspective there is no reason to consider parity a fundamental symmetry. The firld content of QED sufficies to promote parity to an accidental symetry provided a mass scale separation exists between the IR scale, m_e in this case, and any more fundamental UV scale. Indeed, as it turns out, in the SM, which is the more fundamental theory underlying QED, parity is not even approximately a symmetry, and the above effects all exist [a].

The simplicity (and elegance) of particle physics, its being described by simple renormalizable lagrangians, its global symmetries is often associated with its being fundamental. However, from the EQFT perspective, the view is quite the opposite! Simplicity is the unavoidable consequence of our

[a]Although, as we will illustrate later on, while $\psi\bar{\gamma}^\mu\psi\psi\bar{\gamma}_5\gamma_\mu\psi$ is generated by Z boson exchange at tree level, the electric dipole moment arises at high loop order and is thus further suppressed. The cancellation of the lower loop contributions to the edm results from a very specific property of the SM, which in turn could in principle be interpreted as arising accidentally provided the SM is an effective theory with a high cut-off scale.

studying QFT with long wavelength quanta, which blurr short distance details. That is the same simplicity that arises when we look at an object from very far away: to lowest approximation it looks like a point.

2.2. *Symmetries, Selections Rules & Naturalness*

In QFT we distinguish two classes of symmetries:

- *Gauge Symmetries.* These are redundancies in the field description. Their purpose is to allow for a manifestly local description of the interactions of particles with spin ≥ 1. Gauge symmetries can always be viewed as exact: terms explicitly breaking a gauge symmetry in the action can always be viewed as the gauge fixed form of gauge invariant terms. That shows that "gauge symmetry breaking" has more to do with the addition of new degrees of freedom than with the breaking of a symmetry. Massive gauge fields are a classic illustration of that.
- *Global Symmetries.* These can in principle be fundamental, but there is no need for that. As a matter of fact string theory and quantum gravity abhor exact ($\equiv$ fundamental) global symmetries. Moreover global symetries can plausibly arise as accidental and approximate features of a low energy description. One main mechanism for that was illustrated in the previous section. Given there is no fundamental need for global symmetries, given they can easily arise as approximate and accidental, and given quantum gravity abhors them it seems fair to view them as accidental and approximate under all circumstances. This limitation, however, does not reduce their phenomenological importance.

Global symmetries, even when approximate, play a crucial role in the description of the physical world. The core of that role is represented by *selection rules*: the set of group theoretic rules that constrain the way small symmetry breaking parameters can affect observables. Selection rules are at the basis of the notion of *naturalness* which drives much of our speculations beyond the SM, but they are also equally important in more "established" domains like atomic and nuclear physics. The standard examples are in atomic physics where rotational invariance and parity ($SO(3) \times P$) control static quantities, such as level mixing in a external electric field, and dynamical quantities, such as n-photon transitions among levels. Consider for instance the case of an atom placed in a constant electric field whose Hamiltonian is approximated by

$$H = H_0 - \vec{d} \cdot \vec{E} \equiv H(\vec{E}) \tag{3}$$

where H_0 is the Hamiltonian in vacuo and $\vec{d}$ is the atomic electric dipole operator. Eigenvalues and eigenvectors of $H(\vec{E})$ are covariant functions of $\vec{E}$, transforming in a specific way under the action of a rotation g:

$$\lambda_\alpha|\alpha, \vec{E}\rangle = H(\vec{E})|\alpha, \vec{E}\rangle \quad \Rightarrow \quad \lambda_\alpha U_g|\alpha, \vec{E}\rangle = H(R_g\vec{E})U_g|\alpha, \vec{E}\rangle \qquad (4)$$

where U_g and R_g represent g in respectively the Hilbert space and physical 3D space, and where we used $U_g\vec{d}U_g^\dagger = R_g^{-1}\vec{d}$. In practice $U_g|\alpha, \vec{E}\rangle = |\alpha, R_g\vec{E}\rangle$. The mixing among levels with definite angular momentum, which measures the explicit breaking of rotational invariance, can be parametrized by the quantity

$$\mathcal{M}^{JJ_3,J'J'_3}(\alpha, \vec{E}) \equiv \langle J, J_3|\alpha, \vec{E}\rangle\langle\alpha, \vec{E}|J', J'_3\rangle \qquad (5)$$

which measures the product of the probability amplitudes for finding $|J', J'_3\rangle$ and $|J, J_3\rangle$ within the same eigenstate $|\alpha, \vec{E}\rangle$. Using the defining property of the angular momentum eigenstates $U_g^\dagger|J, J_3\rangle = R_{J,g}^{J_3\tilde{J}_3}|J, \tilde{J}_3\rangle$, where ($R_{J,g}$ is the sping J irrep of $SO(3)$) and Eq. (4) we have

$$R_{J,g}^{J_3\tilde{J}_3}R_{J',g}^{J'_3\tilde{J}'_3}\mathcal{M}^{J\tilde{J}_3,J'\tilde{J}'_3}(\alpha, \vec{E}) = \mathcal{M}^{JJ_3,J'J'_3}(\alpha, R_g\vec{E}) \qquad (6)$$

The left hand side transforms like $J \otimes J' = |J-J'| \oplus (|J-J'|+1) \oplus \ldots$. In order to match the same transformation property on the right hand side, $\mathcal{M}^{J\tilde{J}_3,J'\tilde{J}'_3}(\alpha, \vec{E})$ should behave like $|\vec{E}|^{|J-J'|}$ or like an even higher power at small $\vec{E}$: mixing among levels of widely different angular momentum satisfies a definite pattern of suppression at small $\vec{E}$. This is the selection rule[b]. Indeed generically one would indeed expect $\mathcal{M}^{J\tilde{J}_3,J'\tilde{J}'_3}(\alpha, \vec{E}) \sim |\vec{E}|^{|J-J'|}$, and given a system where the behaviour is controlled by a higher power of $\vec{E}$ we would be faced with two options: either we missed other selection rules from an additional symmetry (for instance parity) or some unexplained accidents. Of course numerical accidents can happen when one can choose among a variety of systems with varying fundamental parameters: at some specifically tuned value of those parameters one can have features that are not just simply accounted by selections rules. This possibility of tuning is also what makes the systems in condensed matter interesting. Our practice in that domain of physics suggests that unoccounted tunings plausibly underly a freedom of choice in the fundamental parameters, a landscape of possibilities.

[b]It should be noted that in general the interaction Hamiltonian with also involve terms scaling like $\vec{E}n$ and associated with the corresponding higher mutipoles. The addition of these terms obviously does not affect the conclusion which simply relies on the $SO(3)$ transformation properties of tensor products of $\vec{E}$

The case of the atom in the extermal electric field is also presenting another interesting aspect of symmetry breaking. As long as the electromagnetic field is not excited we can treat $\vec{E}$ as an external non-dynamical parameter *explicitly* breaking $SO(3)$. However more fundamentally $\vec{E}$ is a dynamical field with a non-trivial $SO(3)$ breaking background expectation value. In this respect the breaking of the symmetry is *spontaneous*. The consequences for our discussion are of course the same. In general the parameters breaking a symmetry can be pictured as background expectation values of fictitious fields, *spurions*, with suitable symmetry transformation properties. That is the picture that is normally employed in QFT to draw the consequences of selection rules. For instance, in the SM the selection rules of approximate chiral symmetry are enforced by viewing the Yukawa couplings as fields transforming under the flavor group.

The notion of naturalness arises by considering symmetries and selection rules in QFT. To be concrete, consider a QFT in the regime $E \ll \Lambda$ where we can treat its irrelevant couplings as small perturbations. As a matter of fact to make things even simpler imagine that the theory has a threshold at $\Lambda_{IR} \ll \Lambda$ and that we are looking at physics below Λ_{IR}. The theory will be described by an infinite set of parameters (couplings) $\{\lambda_I\}$. In general it shall be possible to assign to these couplings tranformation properties under all possible symmetries. For instance, one obvious symmetry is dilatation invariance, under which all quantities are rescaled according to their mass dimension. Consider now the expectation value for an observable $\langle \mathcal{O} \rangle$. In general it will also be possible to assign symmetry transformation properties to $\mathcal{O}$. Via quantum fluctuations all allowed combinations of parameters will contribute to $\langle \mathcal{O} \rangle$: in QM mechanics all that is allowed by symmetry is compulsory ("totalitarian principle"). Schematically we will thus have a sum of contributions

$$\langle \mathcal{O} \rangle = \sum_a \langle \mathcal{O} \rangle_a \qquad \langle \mathcal{O} \rangle_a = c_a \lambda_1^{n_{1a}} \dots \lambda_N^{n_{Na}} \tag{7}$$

where c_a are numerical coefficients, and where the combinations of coupling defining each $\langle \mathcal{O} \rangle_a$ precisely matches the symmetry transformation property of $\mathcal{O}$[c]. The constraints imposed by this matches are our selection rules. Again the simplest and more familiar example is provided by the matching of mass dimensions: dimensional analysis. In a situation where the experimentally measured value $|\mathcal{O}_{exp}|$ turns out $\ll \max |\mathcal{O}_a|$ we would

[c]The simple polynomial behavior depicted here is a simplification applying only in specific cases. In general the functional form can be more complicated. The conclusions are however unaffected.

strongly suspect that our descriptions misses some structural aspects. The simplest possibility is that some additional symmetry correlating different $\mathcal{O}_a$ contributions has just not been taken into account. Lacking that option, the situation is just deemed *un-natural*.

$$\text{Un-Naturalness} \quad \equiv \quad \text{failure of selection rules}$$

Within particle physics, the low energy phenomenology of the SM offers some classic examples of naturalness. Two interesting examples are given by the mass differences in the pion system and in the kaon system. Consider $m^2_{\pi^+} - m^2_{\pi_0}$ for illustration. In order to do that it is sufficient to focus on two flavor QCD, which is well described by 4 parameters: Λ_{QCD}, α_{EM}, m_u, m_d. The relevant symmetry is $SU(2)_L \times SU(2)_R$, spontaneously broken to the diagonal isospin $SU(2)$, under which the pion field π_a (a=1,2,3) transforms according to

$$U \equiv e^{i\hat{\pi}} \to V_L U V_R^\dagger = e^{i\hat{\pi}'} \qquad \hat{\pi} = \pi_a \sigma_a \tag{8}$$

The electromagnetic covariant derivative has the form

$$D_\mu U = \partial_\mu U - ieA_\mu(Q_L U + U Q_R) \qquad Q_L = -Q_R = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} \tag{9}$$

so that eQ_L and eQ_R can be promoted to spurions transforming like

$$eQ_L \to V_L eQ_L V_L^\dagger \qquad eQ_R \to V_R eQ_R V_R^\dagger \tag{10}$$

while the quark mass matrix can be assignet the tranformation

$$M_q = \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} \to V_R M_q V_L^\dagger \,. \tag{11}$$

According to these trasformation properties and dimensional analysis the effective pion lagrangian obtained by integrating out quantum fluctuations at the QCD scale $\Lambda_S \sim 1\,\text{GeV}$ will have the form (at lowest order in an expansion in the derivatives and in the spurions)

$$\begin{aligned} \mathcal{L}_{eff} = \frac{f_\pi^2}{4} \Bigg\{ & \text{Tr}\,(\partial U \partial U^\dagger) + a\Lambda_S \text{Tr}\,(M_q U + \text{h.c.}) \\ & - b\Lambda_S \text{Tr}\,\left[(M_q U)^2 + \text{h.c.}\right] - 2c\Lambda_S^2 \frac{e^2}{16\pi^2} \text{Tr}\,(Q_L U Q_R U^\dagger) + \dots \Bigg\} \end{aligned} \tag{12}$$

where, according to Naive Dimensional Analysis (NDA)[d], the coefficients a, b, c are expected to be $O(1)$. By expanding U in a power series in π we

[d]NDA provides an improved form of dimensional analysis whereby factors of $1/4\pi$ are automatically accounted for. This factor, for instance, accounts for the difference between the electromagnetic squared coupling e^2 appearing in the lagrangian and the correspond-

find the leading order mass relation

$$m_{\pi^+}^2 = m_{\pi^0}^2 = \frac{a}{2}\Lambda_S(m_u + m_d) \tag{14}$$

and the higher order splitting

$$m_{\pi^+}^2 - m_{\pi^0}^2 = b(m_u - m_d)^2 + c\frac{e^2}{16\pi^2}\Lambda_S^2 \tag{15}$$

The above formula is an explicit incarnation of Eq. (7). Given the values of m_u, m_d, e^2, Λ_S deduced from other observables, and assuming $b, c = O(1)$, it provides a good account of $m_{\pi^+}^2 - m_{\pi^0}^2$. In particular no cancellation occurs among the two independent contributions, and the second one dominates: the mass difference among pions is dominated by electromagnetic effects. Moreover QCD sum rules associated with unitarity imply $c > 0$, consistent with the intuitive expectation that electrostatic energy is positive for π_+ (a bound state of quarks carrying same sign charges) and negative for π_0 (a bound state of quarks carrying opposite sign charges). We should emphasize that the second term in Eq. (15) can equivalently be estimated by considering the 1-loop correction to the pion self-energy shown in Fig. In the low energy effective theory, scalar electrodynamics, the diagram is quadratically divergent. By cutting off the loop integral at virtuality $p^2 \sim \Lambda_S^2$ we reproduce the estimate Eq. (15). That is always the case: the couplings appearing in the loop diagram and the cut-off automatically realize the same selection rules that inform the top down estimate in Eq. (15).

2.2.1. *The Naturalness Criterion*

Given an EFT with physical cut-off scale Λ_{UV}, we can view its couplings $\{\lambda_I\}$ themselves as *observables* depending on the parameters of a more fundamental description valid at energies $\gg \Lambda_{UV}$. It does then make sense to ask if a given pattern in the set $\{\lambda_I\}$, in particular a pattern of small sizes, can be accounted for by the selection rules of some symmetry. If it can, according to the logic of the previous section, we then say the theory is *natural*. This is 't Hooft's Naturalness Principle (NP) [2], often stated as

ing loop expansion parameter $e^2/(4\pi)^2$ which more appropriately describes the strength of the interaction. The different normalization between couplings and loop expansion parameters is simply associated with the $\hbar = 1$ normalization of natural units. As an exercise, one can easily check that in units where $\hbar = 16\pi^2$ for which the path integral is weighted by

$$e^{\frac{i}{16\pi^2}\int \mathcal{L}} \tag{13}$$

the couplings appearing in $\mathcal{L}$ roughly coincide with the loop expansion parameters. The application of NDA is based on this simple remark [1].

a naturally small parameter λ is always associated with an approximate symmetry becoming exact in the limit $\lambda \to 0$

It is worth illustrating the NP via some simple examples
Example 1. Consider the theory of one scalar ϕ and two Weyl fermions ν_1 and ν_2 with Lagrangian

$$\mathcal{L} = (\partial\phi)^2 + i\bar{\nu}_a \not{\partial} \nu_a + \lambda_{ab}\phi\nu_a\nu_b + \dots \tag{16}$$

Now, given non-vanishing λ_{12} and λ_{22}, one could ask: how small can λ_{11} be tolerated according to the NP. To answer this question we must consider the $U(1)_1 \times U(1)_2$ flavor symmetry under which field and couplings (viewed as spurions) have charges Scaling dimension and $U(1)_1 \times U(1)_2$ charges are

	ϕ	ν_1	ν_2	λ_{11}	λ_{12}	λ_{22}
$U(1)_1$	0	1	0	-2	-1	0
$U(1)_2$	0	0	1	0	-1	-2

thus matched according to

$$[\lambda_{11}] = [\lambda_{12}^2\lambda_{22}^*] \tag{17}$$

Suggesting that whatever microscopic source gives rise to small λ_{12} and λ_{22} should also give rise to $\lambda_{11} \propto \lambda_{12}^2\lambda_{22}^*$. Indeed RG evolution from Λ_{UV} down to a scale μ in the effective theory precisely gives rise to such a contribution through the diagram in Fig

$$\delta\lambda_{11}(\mu) \sim \frac{\lambda_{12}^2\lambda_{22}^*}{16\pi^2} \ln \Lambda_{UV}/\mu \,. \tag{18}$$

According to naturalness the above equation represents the smallest value of λ_{11} compatible with naturalness. In particular at $\mu \sim \Lambda_{UV}$ the minimal value according to naturalness is $\lambda_{11} \sim \lambda_{12}^2\lambda_{22}^*/16\pi^2$.
Example 2. Consider now the theory of two real scalar fields

$$\mathcal{L} = (\partial\phi_1)^2 + (\partial\phi_2)^2 + g_{11}\phi_1^4 + g_{12}\phi_1^2\phi_2^2 + g_{22}\phi_2^4 \tag{19}$$

and ask how small can g_{11} be tolerated given $g_{12}, g_{22} \neq 0$. The scalar fields are real and admit no phase rotations. However another class of symmetries plays a role here. These are the infinite symmetries enjoyed by free field theory. Consider indeed a massive free scalar ϕ, whose action can be written in spacetime and Fourier space as

$$S = \int d^4x \phi(x)(-\partial^2 - m^2)\phi(x) = \int \frac{d^4p}{(2\pi)^4}\hat{\phi}(-p)(p^2 - m^2)\hat{\phi}(p) \tag{20}$$

where $\hat{\phi}(-p) = \hat{\phi}(p)^*$. It is evident from the Fourier space representation that the transformation

$$\hat{\phi}(p) \to e^{i\theta(p)}\hat{\phi}(p) \qquad\qquad \theta(-p) = -\theta(p) \tag{21}$$

is a symmetry of the action. By expanding θ in a power series $\theta = a_\mu p^\mu + a_{\mu\nu\rho} + \dots$ the transformation in position space reads

$$\phi \to (1 + a_\mu \partial^\mu - a_{\mu\nu\rho}\partial^\mu\partial^\nu\partial^\rho + \dots)\phi \tag{22}$$

for which the Noether curents are the tensor bilinears in ϕ with even rank $r \geq 2$ (the energy momentum tensor corresponding to $r = 2$). Consider now the 1PI vertex for n fields in Fourier space $\Gamma^{(n)}(p_1, \dots, p_n)$. It can formally be assigned the transformation property

$$\Gamma^{(n)}(p_1, \dots, p_n) \to \Gamma^{(n)}(p_1, \dots, p_n)e^{-i\sum_{a=1}^{a=n}\theta(p_a)}\,. \tag{23}$$

For a general θ satisfying Eq. (21), invariance of the effective action then implies that only the two point function can be non vanishing and moreover of the form $\Gamma^{(2)} = \delta^{(4)}(p_1+p_2)\Gamma(p_1)$. This peculiar symmetry thus protects all vertices with more than two legs, that is all non trivial interactions.

In the problem at hand we can assign independent tranformation properties controlled by phases $\theta_1(p)$ and $\theta_2(p)$ for respectively $\hat{\phi}_1(p)$ and $\hat{\phi}_2(p)$. In Fourier space the couplings in Eq. (19) can be extendended to general (complex) spurions with transformation property

$$g_{11}(p_1, p_2, p_3, p_4) \to g_{11}(p_1, p_2, p_3, p_4)e^{-i[\theta_1(p_1)+\theta_1(p_2)+\theta_1(p_3)+\theta_1(p_4)]} \tag{24}$$
$$g_{12}(p_1, p_2, p_3, p_4) \to g_{12}(p_1, p_2, p_3, p_4)e^{-i[\theta_1(p_1)+\theta_1(p_2)+\theta_2(p_3)+\theta_2(p_4)]} \tag{25}$$

implying the following matching of quantum numbers

$$[g_{11}(p_1, p_2, p_3, p_4)] = [g_{12}(p_1, p_2, p_5, p_6)g_{12}(p_3, p_4, -p_5, -p_6)]\,. \tag{26}$$

This suggests $g_{11} \sim g_{12}^2$ for the minimal natural value of g_{11}. Once again this deduction coincides (modulo trivial 4π factors) with the 1-loop renormalization of g_{11} induced by the diagram in Fig.

Example 3. Consider a theory of Dirac spinors charged under two gauged $U(1)'s$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu}^{(1)})^2 - \frac{1}{4}(F_{\mu\nu}^{(2)})^2 + i\bar{\Psi}\gamma^\mu(\partial_\mu - ie_1Q_1A_\mu^{(1)} - ie_2Q_2A_\mu^{(2)})\Psi \tag{27}$$

where $Q_{1,2}$ are matrices with $O(1)$ entries. As an exercise one could estimate how big in such a theory one can tolerate a mixing $k_{12}F_{\mu\nu}^{(1)}F^{(2)\mu\nu}$ in the action.

Example 4. Consider the theory of one scalar field

$$\mathcal{L} = (\partial\phi)^2 - m^2(\phi)^2 - \lambda\phi^4 + \ldots \,. \tag{28}$$

Let us also assume this theory to be an effective one valid below a *physical* UV cut-off Λ_{UV}. For instance we could imagine ϕ to be a composite from some dynamics characterized by the scale Λ_{UV}. We could then ask how large m^2 can be tolerated given the value of λ and Λ_{UV}. There are various ways to proceed according to symmetries. One is to use the same logic of example 2: λ can be viewed as a spurion $\hat{\lambda}(p_1, p_2, p_3, p_4)$ transforming with the analogue of Eq. (24) under the symmetry $\hat{\phi}(p) \to e^{i\theta(p)}\hat{\phi}(p)$. Selection rules then allow a contribution to the 2-point function $\delta\Gamma(p, -p) \propto \lambda(p, -p, k, -k)\Lambda_{UV}^2 \to \lambda\Lambda_{UV}^2$. Indeed the loop diagram in figure gives

$$\delta m^2 \sim \frac{\lambda}{16\pi^2}\int dk^2 \sim \frac{\lambda}{16\pi^2}\Lambda_{UV}^2 \,. \tag{29}$$

A more standard argument uses the shift symmetry $\phi \to \phi + c$, which protects the mass term but which is broken by the quartic coupling λ: quantum effects proportional to λ then allow the generation of a mass term. A similar argument can be carried through for a scalar ϕ coupled to a fermion via a Yukawa coupling $y\phi\bar{\psi}\psi$, where one then expects $\delta m^2 \sim (y^2/16\pi^2)\Lambda^2$. This latter case is the one the matters in the SM with the role of ϕ and ψ played respectively by the Higgs scalar h and by the top quark t. These results are at the basis of the famous bias against the existence of interacting light scalars in QFT [3, 4].

A parameter choice that can be accounted for by selection rules seems clearly more plausible than a choice that cannot, but such plausibility does not represent yet an explanation. It does however set the stage for a deeper explanation: an approximate symmetry of an effective lagrangian (even one affecting its relevant couplings) can be explained as an accidental symmetry in an even more fundamental description. A standard example of that is given by the neutrino mass in the Fermi-Cabibbo theory that predates the SM. Such an effective theory, consists of Dirac charged leptons and their left handed neutrino partners, interacting via 4-fermion terms and with electromagnetism as the only gauge interaction. In such a theory, gauge invariance allows for a Majorana neutrino mass. However such a relevant mass parameter can be taken as naturally small (ot even to vanish) by postulating the fundamental theory respects a global symmetry, lepton number. The assumption of lepton number "explains" why neutrinos are so ligh but no deeper explanation for the why's of such a global symmetry can be given in the Fermi-Cabibbo theory. A deeper explanation instead nicely comes in the SM, where lepton number arises as an accidental symmetry of the relevant and marginal couplings, while it is violated by dimension 5 operators

$$\mathcal{L}_{\Delta L=2} = \frac{(H\ell)^2}{\Lambda_{UV}}\,. \tag{30}$$

Provided a separation of mass scale exists, approximate lepton number is economically explained as accidental symmetry. On one hand this illustrates once more the central relevance of hierarchical separation of mass scales to account for the structure of particle interactions. On the other it suggests the slogan: "Today's approximate symmetries are tomorrow's accidental ones".

While the naturalness principle offers a guideline to interpret the structure of fundamental interactions, one should be aware of its possible limitations. The most obvious one is associated with the notion of "landscape", which is to say a manifold of similar systems spanned by continuum or near continuum changes of parameters. The variety of condensed matter systems that can be concocted in the lab is an excellent example of that. Such systems feature critical points, which essentially correspond to fine tuned choices of the parameters where naturalness fails. The standard example is given by second order phase transitions, chracterized by the vanishing of the "mass" of some interacting bosonic degree of freedom. The generic condensed matter systems do satisfy naturalness, but for our human (antropic) reasons, applications and fun study, we often pick those few who do not. The same reasoning can be exported to fundamental particle physics: there could exists a manifold of vacua corresponding to a lanscape of effective field theories, where antropic selection through cosmological evolution picks critical points in parameter space, where naturalness fails. We shall comment more about this whne discussing the hierarchy problem.

Another, perhaps less dramatic, but equally intriguing limitation of naturalness concerns the expectation that any global symmetry must be explicitly broken by quantum gravity effects. The unavoidable amount of this breaking is however not easily pinpointed in general. One slightly more quantitative expectation associated with quantum gravity is given by the "weak gravity conjecture" (WGC). According to the broad WGC there cannot exist a consistent EFT where there exist interactions that are weaker than gravity within the domain of validity of the EFT. Strickly speaking WGC was formulated for interactions medited by forms, $U(1)$ gauge fields in particular, and checked to be valid in all possible EFTs that descend from string theory. The conflict with naturalness is clear. We have seen that, given the infinite symmetries of free field theory, any field theory can be naturally made arbitrarily weakly coupled. However for sufficiently weak effective couplings the WGC is violated, showing that naturalness is not enough to assess the plausibility of a given structure. It should however be noticed that, aside the landscape, we have no convincing story for the reversed situation, where small parameters apepar not associated to symmetries.

2.3. *Natural and un-Natural mass hierarchies*

Consider again the standard EFT situation depicted in Fig. The IR scale Λ_{IR} is a function of the parameters $\{\lambda_I\}$ describing the deformed CFT as it emerges at the UV scale Λ. In this situation the hierarchy $\Lambda_{IR} \ll \Lambda$ may or may not be natural. For instance, Example 4 in the previous subsection shows that in the effective theory of a scalar field ϕ, the choice $\Lambda_{IR}^2 \equiv m^2 \ll \lambda\Lambda^2/16\pi^2$ is unnatural.

The emergence of Λ_{IR} can be pictured from the perspective of the RG-flow, see Fig. The basic picture is this: the flow starts near a fixed point at a renormalization scale $\mu_{RG} \sim \Lambda$ while the scale Λ_{IR} corresponds the RG scale where the deformation becomes of order 1. One can conveniently choose coordinates $\{\lambda_I\}$ such that the fixed point is at $\lambda_I = 0$ and such that when some λ_I becomes $O(1)$ the correlators of the original theory are perturbed by $O(1)$. Normally the latter coincides with the point where the the β functions $\beta_I \sim O(1)$, so that the matrix elements of the energy momentum trace $T_\mu^\mu = \beta_I \mathcal{O}_I$ cannot be treated as small. The corresponding $O(1)$ breakdown of scale invariance is one way to describe the emergence of a physical scale, Λ_{IR}. In this picture, a hierarchy of scales $\Lambda_{IR}/\Lambda \ll 1$, corresponds to a slow RG evolution. Consequently a natural hierarchy corresponds to a naturally slow RG evolution, i.e. to the natural stability of the original fixed point. The two canonical options to achieve a stable fixed point are respectively based on *Marginality* and *Symmetry.*

2.3.1. *Natural Hierarchy from Marginality*

This is the situation where the fixed point for some structural reason does not possess any strongly relevant deformation and possesses instead only marginally relevant deformations. In That is to say there are no scalar operators of dimension strictly less than 4, such as mass terms in weakly coupled theories, and the only scalars have dimension near 4. The simplest example in this class is given by Yang-Mills theory, for which the leading deformation, the gauge kinetic term, is marginally relevant and corresponds to the gauge coupling. Assuming the gauge group is $SU(N)$ the strength of the deformation is well parametrized by $\lambda = 11g^2N/48\pi^2$ for which we have the RG evolution

$$\mu\frac{d\lambda}{d\mu} = -2\lambda^2 \qquad \Longrightarrow \qquad \lambda(\mu) = \frac{\lambda_{UV}}{1 + 2\lambda_{UV}\ln\frac{\mu}{\Lambda}}\,. \tag{31}$$

λ is the parameter that controls the loop expansion and violation of scale invariance: $\lambda \sim 1$ thus corresponds to the IR scale Λ_{IR}

$$\lambda(\Lambda_{IR}) \sim 1 \qquad \Longrightarrow \qquad \frac{\Lambda_{IR}}{\Lambda} \sim e^{-1/\lambda_{UV}}\,. \tag{32}$$

This is the well known dimensional-trasmutation phenomenon of non abelian gauge theories. The UV coupling λ_{UV} is traded for the strong coupling scale Λ_{IR}, which moreover arises non-perturbatively: it is enough an *algebraically small* value of λ_{UV} (say 1/20) to produce a *hierarchically* small Λ_{IR}/Λ.

A qualitatively similar conclusion is reached for the case in which the deformation of lowest dimension is only slightly relevant and associated with a scalar primary of dimension $\Delta = 4 - \epsilon$ [5–7]

$$\mathcal{L}_{mass} = c\Lambda^\epsilon \mathcal{O}_{4-\epsilon} \qquad\qquad 0 < \epsilon \ll 1\,. \tag{33}$$

One is easily convinced that the above term induces deformations controlled by effective runnning coupling $\lambda(E) \equiv c(\Lambda/E)^\epsilon$. This can for instance be seen by considering the correction to a generic 2-point function $\langle\mathcal{O}'(x)\mathcal{O}'(0)\rangle = 1/x^{2\Delta'}$ using conformal perturbation theory. One has

$$\delta\,\langle\mathcal{O}'(x)\mathcal{O}'(0)\rangle = c\Lambda^\epsilon \int d^4z \langle\mathcal{O}'(x)\mathcal{O}'(0)\mathcal{O}(z)\rangle = \tag{34}$$

$$c\Lambda^\epsilon \int d^4z \frac{C_{\mathcal{O}'\mathcal{O}'\mathcal{O}}}{x^{2\Delta'-4+\epsilon}(z-x)^{4-\epsilon}z^{4-\epsilon}} \sim \frac{C_{\mathcal{O}'\mathcal{O}'\mathcal{O}}c(\Lambda x)^\epsilon}{x^{2\Delta'}} \tag{35}$$

so that in a normalization where $C_{\mathcal{O}'\mathcal{O}'\mathcal{O}} = O(1)$, the relative size of the deformation is indeed $\lambda(1/x) \equiv c(\Lambda x)^\epsilon$. According to our previous discussion Λ_{IR} is therefore given by

$$\lambda(\Lambda_{IR}) \sim 1 \quad \Longleftrightarrow \quad \Lambda_{IR} = c^{1/\epsilon}\Lambda\,. \tag{36}$$

Again we find that *algebraically small* parameters, say $c \sim \epsilon \sim 1/10$ can seed a hierarchically small mass scale [e]. The scenario we just outlined

[e]We limited our discussion to leading order effects in λ. That coincides to taking $\mu d\lambda(\mu)/d\mu \equiv \beta(\lambda) = -\epsilon\lambda(\mu)$, while in general we expect $\beta = -\epsilon\lambda + a_2\lambda^2 + a_3\lambda^3 + \dots$. For instance, in conformal perturbation theory the OPE expansion $\mathcal{O}_{4-\epsilon}(x)\mathcal{O}_{4-\epsilon}(0) = \dots + c_2x^{\epsilon-4}\mathcal{O}_{4-\epsilon}(0) + \dots$ implies $a_2 \propto c_2$. One can easily check that the resulting hierarchy is quantitatively different though the conclusions are qualitatively the same.

corresponds to the dual description of the Randall-Sundrum [8] model with Goldberger-Wise [9]stabilization of the volume modulus [10], the radion. In that case the quasi marginal operator $\mathcal{O}_{4-\epsilon}$ is dual to a 5D scalar with mass $m^2 = -\epsilon/L^2$ where L is the AdS radius.

2.3.2. *Natural Hierarchy from Symmetry*

This is the situation where the fixed point does possess strongly relevant deformations, but they can all be associated with the breaking of global symmetries. The corresponding coefficients can thus be naturally assumed to be small. The hierarchy controlled by these coefficients is therefore natural. In other words the fixed point is a point of enhanced symmetry, so that it is technically natural to imagine the theory to emerge from the UV sitting extremely close to it, in such a way that it takes a long RG time to exit the neighbourhood of the fixed point. The simplest example of this general situation is given by QCD with N_F quark flavors

$$\mathcal{L} = -\frac{1}{4g^2}F^2_{\mu\nu} + i\bar{\psi}\not{D}\psi - \bar{\psi}_L\hat{M}\psi_R - \bar{\psi}_R\hat{M}^\dagger\psi_L \tag{37}$$

where $\hat{M}$ is the $N_F \times N_F$ mass matrix. The system possess a chiral symmetry $SU(N_F)\times SU(N_F)$ explicit broken by the mass matrix, which formally transforms according to $\hat{M} \to U_L\hat{M}U_R^\dagger$. At weak coupling the theory sits near the free fixed point, where the only relevant operator is fermion bilinear $\bar{\psi}\psi$, while the gauge kinetic term is marginal (relevant or irrilevant depending on N_F/N_c). If we can consistently assume that the theory at the UV scale Λ, where the above theory emerges, respects approximately chiral symmetry, then the choice $|\hat{M}| \ll \Lambda$ is natural, as the hierarchy of scales it implies.

As we already mentioned, approximate symmetries not only offer plausible and self-consistent parameter choices but also set the stage for a deeper explanation. For instance, in the case we just illustrated the chiral symmetry may arise as accidental and approximate at the scale Λ. The typical situation where that happens is one where above the scale Λ one has a gauge theory where ψ sits in a chiral representation such that a mass term is not gauge invariant and such that the lowest dimensional interaction breaking chiral symmetry is of the form

$$\mathcal{L}_{mass} = \frac{\bar{T}T\bar{\psi}_L\hat{Y}\psi_R}{{\Lambda'_{UV}}^2} \tag{38}$$

where T is another fermion charged under a strong gauge group G that confines at the scale Λ, where $\hat{Y}$ is a $N_F \times N_F$ and where Λ'_{UV} is a UV threshold much above Λ. Now, the gauge dynamics gives rise to a condensate $\langle \bar{T} t\rangle \sim \Lambda^3$ so that in the low-energy theory ψ has an effective mass $\hat{M} \sim \hat{Y}\Lambda^3/{\Lambda'_{UV}}^2 \ll \Lambda$. Of course one could go on and ask what gave rise to the hierarchy $\Lambda \ll \Lambda'_{UV}$. For instance that hierarchy may arise by the mechanism of marginality we illustrated in the previous section, which does not rely on approximate symmetries, but just on numerical accidents. What we have just illustrated is the origina of fermion masses in Technicolor extensions of the SM. Such an extension would work well if all the SM fermions were sufficiently light. But that is unfortunately not the case, for the top quark in particular.

Another example of a symmetry protecting a hierarchy is given by supersymmetry. Here one can envisage a situation where in the limit of unbroken supersymmetry (and normally assuming some additional chiral symmetry, like the $U(1)_{PQ}$ in the minimal supersymmetric model, as well discuss later on) no strongly relevant operators are permitted. Very schematically we thus have a lagrangian

$$\mathcal{L} = \mathcal{L}_{SUSY} - \tilde{m}^2 \tilde{f}^* \tilde{f} \tag{39}$$

where the second term is a relevant supersymmetry breaking mass term for the sfermions. Again it is techically natural to assume $\tilde{m}^2 \ll \Lambda^2$ as the mass term breaks supersymmetry explicitly. While this choice is natural there seems to be no a priory reason for it, but again it turns out this stae of things sets the stage for a deeper explanation. This explanation comes when one assumes the breaking of supersymmetry is indeed spontaneous. It is a fact that such breaking either arises at tree level or non-perturbatively. If, for some structural reason, supersymmetry is unbroken at tree level, then the scale of supersymmetry breaking must be related to the fndamental scale via a relation of the type

$$\tilde{m}^2 \sim e^{-1/\lambda}\Lambda^2 \tag{40}$$

where λ is some loop expansion parameter (coupling). Like in section it is sufficiently an moderetely weak interaction to generate a hierarchical separation of scales.

2.4. *The Standard Model and the Hierarchy Paradox*

We would now like to analyze the structure of the Standard Model in the light of the ideology developed in the previous sections. Our view can therefore be summarized as follows

- The SM is an EFT valid below some physical cut-off scale Λ (this scale could concretely correspond to the mass scale of superpartners, to the GUT scale or to the string scale)
- Given the SM gauge group and field content we write all possible terms
- Global symmetries may, or may not, be invoked.

The details of the SM field content are well known and we shall not linger on them. We shall often indicate collectively with $F_{\mu\nu}$ the field strength of its gauge group $G_{SM} \equiv SU(3) \times SU(2) \times U(1)_Y$. Similarly we shall indicate the Higgs field by H and by Ψ the matter fields consisting of Weyl fermions in three families of leptons ℓ, e_c and quarks q, u_c, d_c. We recall the quantum numbers under G_{SM}

$$\begin{array}{ll} \ell\ = (\mathbf{1}, \mathbf{2}, -1) & \qquad q\ = (\mathbf{3}, \mathbf{2}, 1/3) \\ e_c = (\mathbf{1}, \mathbf{1}, 2) & \qquad u_c = (\bar{\mathbf{3}}, \mathbf{1}, -4/3)\ . \\ H = (\mathbf{1}, \mathbf{2}, 1) & \qquad d_c = (\bar{\mathbf{3}}, \mathbf{1}, 2/3) \end{array} \tag{41}$$

We shall indicate the gauge indices only when really needed, while we indicate the components of $SU(2)$ doublet by

$$\ell \equiv \begin{pmatrix} \nu \\ e \end{pmatrix} \qquad\qquad q \equiv \begin{pmatrix} u \\ d \end{pmatrix} \tag{42}$$

We could complement our discussion by making the spacetime metric $g_{\mu\nu}$ dynamical and promoting our lagrangian to the most general one invariant under $G_{SM} \times$ diffs. We shall however mostly focus on the non-gravitational interactions.

The SM lagrangian can be organized in a $1/\Lambda$ expansion

$$\mathcal{L} = \sum_d \frac{1}{\Lambda^{d-4}} \mathcal{L}^{(d)} \tag{43}$$

where $\mathcal{L}^{(d)}$ consists of operators of dimension d. For large Λ this expansion is remarkably well matched with many empirical facts, as we shall now briefly illustrate by considering the lowest few terms. The order in which we shall proceed and some slight cheating (which we expose at the end) are part of the rhetoric to make our point.

Consider first $\mathcal{L}^{(4)}$ whose general structure is

$$\mathcal{L}^{(4)} = \frac{-1}{4g^2} F_{\mu\nu} F^{\mu\nu} + i\bar{\Psi} \not{D} \Psi + |DH|^2 + Y_{ij} H \Psi_i \Psi_{cj} + \lambda |H|^4\,. \tag{44}$$

Here, one should stress, each and every term has been observed in Nature. Moreover $\mathcal{L}^{(4)}$ happens to possess unique structural features which are at the heart of its remarkable empirical success. In particular $\mathcal{L}^{(4)}$ is automatically endowed with some crucial accidental symmetries. Consider then $\mathcal{L}^{(5)}$ and $\mathcal{L}^{(6)}$

$$\mathcal{L}^{(5)} = \frac{y^\nu_{ij}}{\Lambda}(\ell_i H)(\ell_j H) \tag{45}$$

$$\mathcal{L}^{(6)} = \frac{k_{ij}}{\Lambda^2}(\ell_i \sigma_{\mu\nu} e_{cj})HB^{\mu\nu} + \frac{k^{(1)}_{ijkl}}{\Lambda^2}(\bar{q}_i \sigma_\mu q_j)(\bar{q}_k \sigma_\mu q_\ell) + \tag{46}$$

$$+ \frac{k^{(2)}_{ijkl}}{\Lambda^2}(u^\alpha_{ci} u^\beta_{cj})(e_{ck} d^\gamma_{c\ell})\epsilon_{\alpha\beta\gamma} + \frac{c_T}{\Lambda^2}(H^\dagger D_\mu H)^2 + \dots \tag{47}$$

where we have written just a few representative terms, and employed a sort of squiggly notation (we explicitly show gauge, color, indices only in the third term, to stress that all quarks in that operator tranform in the fundamental). The crucial remark here is that as soon as one considers operators of dimension $d \geq 5$ one encounters effects which were either never observed or which were found to be "tiny". Let us analize that in some more detail. Consider first $\mathcal{L}^{(5)}$, where we find just one class of operators. Their main phenomenological implication is to endow neutrinos with a mass matrix as soon as H acquires an expectation value

$$m^\nu_{ij} = \frac{y^\nu_{ij} v^2}{\Lambda} \qquad\qquad \langle H \rangle \equiv \begin{pmatrix} 0 \\ v \end{pmatrix} . \tag{48}$$

Notice that $\mathcal{L}^{(4)}$ gives instead an exactly vanishing contribution to neutrino masses. That is because in the absence of right-handed neutrinos the only possible neutrino mass is of Majorana type

$$m_{ij}\nu_j\nu_i \tag{49}$$

and breaks lepton number $\ell_i \to e^{i\alpha}\ell_i$, $e_{ci} \to e^{-i\alpha}e_{ci}$, while $\mathcal{L}^{(4)}$ respects it.

Now neutrino masses are experimentally found to be tiny. Measurements of neutrino oscillations in the sun and in the atmosphere imply differences Δm^ν between mass eigenvalues of respectively $\sim 0.01\,\mathrm{eV}$ and $\sim 0.05\,\mathrm{eV}$. Observation of the large scale structure of the universe, on the other, hand implies an upper bound on the sum of neutrino masses

$$\sum_{i=1}^{i=3} m^\nu_i < 0.23\,\mathrm{eV} \qquad 95\%\mathrm{CL}\,. \tag{50}$$

These observations are nicely accounted for if Λ is very large. For instance if $y^\nu_{ij} \sim O(1)$ then $\Lambda \sim 10^{15} GeV$ implies masses in the right ballpark (i.e. $\sim 0.1\,\mathrm{eV}$). It is also interesting to compare the structure of the spectrum of charged fermions to that of neutrinos. The masses of charged fermion are distributed over roughly 5 orders of magnitude with the heaviest (the top quark) at $\simeq 174\,\mathrm{GeV}$ and the lightest (the electron) at $\simeq 0.511\,\mathrm{MeV}$. Moreover over these 5 orders of magnitude the masses are roughly evenly spaced: within each charge type (up-quarks, down-quarks and charged leptons) eigenvalues have splittings that range between 1 and 2 orders of magnitude. On the other hand below the mass of the electron one finds a gap of about 7 orders of magnitude before finding the neutrino masses and moreover these appear to be less hierarchically split among themselves than the charged masses are. Overall, these observations speak for a qualitatively different origin of charged and neutral fermion masses. The fact that charged and neutral masses arise form respectively marginal and irrelevant operators, gives, provided $\Lambda \gg v$, a nice and simple explanation for the large gap between their masses. Notice that within the old-fashioned pre-Wilsonian definition of the SM as a renormalizable theory, the fact that neutrinos have a mass at all appears as a drawback, given the corresponding operator is shunned by renormalizability. In the modern EFT approach, the fact that neutrinos have a tiny mass compared to all other states is instead viewed as both a structural success and as an indication that the next layer Λ is much above the weak scale. [f]

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[f] This view is simple and suggestive, but there obviously are alternatives. One standard way to generate the operators in $\mathcal{L}_5$ is to add right handed neutrinos, that is Weyl fermions ν_c transforming as total singlets of the SM. The ν_c's (say 3 of them to match the family replication in the other charge sectors) will couple to ℓ_i via Yukawa couplings Y_N and will also in general have a mass (matrix) $\hat{M}$, whose eigenvalues play the role of the cut-off Λ. Integrating our the ν_c's one generates $\mathcal{L}_5$ with $y^\nu_{ij}/\Lambda = \left(Y_N^T \hat{M}^{-1} Y_N\right)_{ij}$ Clearly the small m_ν can also be accounted for by small Yukawa's even for $|\hat{M}| \sim 100\,\mathrm{GeV}$. But notice that in the latter case the overall size of all entries in Y_N should be smaller than the Yukawa of the electron.

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