On the one loop effective action in $6D, \mathcal{N} = (1, 0)$ hypermultiplet self-coupling model

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Motivation

1. The supersymmetric sigma models in six dimensions are an interesting objects of study due to their remarkable properties, namely because of the intimate connection with the differential geometry. They were studied in old works of (G. Sierra and P.K. Townsend, 1983).
2. The aim is to make the generalization of the SUSY σ-model on the harmonic superspace (A.Galperin, E. Ivanov, S. Kalitsyn, V. Ogievetsky, E. Sokatchev, 1985), (A.Galperin, E. Ivanov, V. Ogievetsky, E. Sokatchev, Harmonic Superspace, 2001).
3. We consider the $\mathcal{N} = (1, 0)$ supersymmetric model of interacting q-hypermultiplet and abelian gauge multiplet V^{++} in six-dimensional harmonic superspace. For the vanishing gauge multiplet, this theory describes the supersymmetric sigma-model with general multicenter hyper-Kähler metric (A. Galperin, E. Ivanov, V. Ogievetsky, E. Sokatchev, 1986).
4. Our goal is to calculate the divergences of the model under consideration.

Plan

1. The model
2. One loop effective action
3. Algebra of covariant derivatives
4. One loop divergences
5. Self interaction case
6. Summary

The Model

We use the harmonic superspace formulation for the theory (A.Galperin, E. Ivanov, V. Ogievetsky, E. Sokatchev, Harmonic Superspace, 2001).

$$(z,u) = (x^M, \theta^{ai}, u^{\pm i}), \qquad M = 0,..,5, \quad a = 1,..,4, \quad i = 1,2. \tag{1}$$

The harmonic variables $u^{\pm i}$ parameterize the coset $SU(2)/U(1)$ and obey the constraints $u^{+i}u_i^- = 1$, $u_i^- \equiv (u^{+i})^*$. The harmonic superspace admits the analytic basis $\zeta = (x^M_{\mathcal{A}}, \theta^{\pm a})$

$$x^M_{\mathcal{A}} = x^M + \frac{i}{2}\theta^{+a}\gamma^M_{ab}\theta^{-b}, \quad \theta^{\pm a} = u^{\pm}_k \theta^{ak}, \tag{2}$$

where $(\gamma^M)_{ab}$ are the six-dimensional Weyl matrices.
The action for the model is

$$\begin{aligned} S_0[q^+, V^{++}] \quad &= \quad \frac{1}{4f^2}\int d^{14}z \frac{du_1 du_2}{(u_1^+ u_2^+)^2} V^{++}(z,u_1)V^{++}(z,u_2) \\ &\quad - \int d\zeta^{(-4)}\Big(\tilde{q}^+ D^{++} q^+ + i\tilde{q}^+ V^{++} q^+ + L^{(+4)}\Big), \end{aligned} \tag{3}$$

where the superspace integration measures $d\zeta^{(-4)} = d^6x_{\mathcal{A}} du$, $d^{14}z = d^6x_{\mathcal{A}}\,(D^+)^4(D^-)^4$. The gauge transformations

$$\delta V^{++} = -D^{++}\lambda\,, \quad \delta q^+ = i\lambda\, q^+\,, \quad \delta\tilde{q}^+ = -i\lambda\,\tilde{q}^+\,, \tag{4}$$

hold the action (3) invariant when the potential function for hypermultiplet depends on the invariant combination of superfields, $L^{(+4)} = L^{(+4)}(\tilde{q}^+ q^+)$.

We use the background field method in harmonic superspace ((I.L. Buchbinder, E.I. Buchbinder, S.M. Kuzenko, B.A. Ovrut, 1998) and references therein)

$$V^{++} \to V^{++} + f v^{++}, \qquad q^{+} \to Q^{+} + q^{+}, \tag{5}$$

The one-loop effective action

$$\Gamma^{(1)} = i\,\mathsf{Tr}_{(3,1)} \ln \mathcal{D}^{++} + \tfrac{i}{2}\,\mathsf{Tr}_{(2,2)} \ln \left(\partial^2 - 4f^2 \widetilde{Q}^{+} \mathcal{G}^{(1,1)} Q^{+} + \ldots\right), \tag{6}$$

where we have introduced the covariant harmonic derivative

$$\mathcal{D}^{++} = D^{++} + iV^{++} + i\Psi^{++}, \quad \Psi^{++} = -i\frac{\partial L^{+4}(\tilde{Q}^{+}Q^{+})}{\partial(\tilde{Q}^{+}Q^{+})}, \quad \Psi = \frac{\partial^2 L^{+4}(\tilde{Q}^{+}Q^{+})}{\partial(\tilde{Q}^{+}Q^{+})^2}, \tag{7}$$

and $\mathcal{G}^{(1,1)}$ is the Green function that satisfies the equation

$$\mathcal{D}^{++}\mathcal{G}^{(1,1)}(1|2) = \delta^{(3,1)}(1|2)\,. \tag{8}$$

The functional trace includes the matrix trace and integration over harmonic superspace

$$\mathsf{Tr}_{(q,4-q)}\mathcal{O} = \mathsf{tr} \int d\zeta_1^{(-4)} d\zeta_2^{(-4)}\, \delta_{\mathcal{A}}^{(4-q,q)}(1|2)\, \mathcal{O}^{(q,4-q)}(1|2).$$

We introduce the notation

$$\mathcal{V}^{++} = V^{++} + \Psi^{++}. \tag{9}$$

Then following standard procedure (A.Galperin, E. Ivanov, V. Ogievetsky, E. Sokatchev, Harmonic Superspace, 2001), we consider the non-analytic gauge connection $\mathcal{V}^{--}$ by the rule

$$D^{--}\mathcal{V}^{++} = D^{++}\mathcal{V}^{--}\,, \tag{10}$$

which can be solved exactly in the abelian case

$$\mathcal{V}^{--} = \int du_1 \, \frac{\mathcal{V}^{++}(u_1)}{(u^+u_1^+)^2}\,. \tag{11}$$

Then we rewrite the standard algebra of spinor and harmonic derivatives

$$\begin{aligned}
&[\nabla^{--}, D_a^+] = \nabla_a^-\,, \quad [\nabla^{++}, \nabla_a^-] = D_a^+\,, \quad [\nabla^{++}, D_a^+] = [\nabla^{--}, \nabla_a^-] = 0\,,\\
&[D_a^+, \nabla_b^-] = 2i\nabla_{ab}\,, \quad [D_a^+, \nabla_{bc}] = \tfrac{i}{2}\varepsilon_{abcd}\mathcal{W}^{+d}\,, \quad [\nabla_a^-, \nabla_{bc}] = \tfrac{i}{2}\varepsilon_{abcd}\mathcal{W}^{-d}\,,
\end{aligned} \tag{12}$$

where

$$\mathcal{W}^{+a} = -\tfrac{i}{6}\varepsilon^{abcd}D_b^+D_c^+D_d^+\mathcal{V}^{--}\,, \quad \mathcal{W}^{-a} := \nabla^{--}\mathcal{W}^{+a}\,. \tag{13}$$

The covariant analytic d'Alembertian

$$\widehat{\Box} = \eta^{MN}\mathcal{D}_M\mathcal{D}_N + \mathcal{W}^{+a}\mathcal{D}_a^- + \mathcal{F}^{++}\mathcal{D}^{--} - \tfrac{1}{2}(\mathcal{D}^{--}\mathcal{F}^{++}). \tag{14}$$

The method for calculating the trace of logarithm of first-order differential operator, $\Gamma_q = i\,\mathrm{Tr}\,\ln \mathcal{D}^{++}$ was developed in work (I.L. Buchbinder, S.M. Kuzenko, 1998).
We also use the proper-time representation for the operator $\widehat{\Box}^{\,-1}$

$$\frac{1}{\widehat{\Box}} = \int_0^\infty d(is)(is\mu^2)^{\frac{\varepsilon}{2}} e^{-is\widehat{\Box}} \tag{15}$$

in the Green function with such a formal expression (I.L. Buchbinder, N.G. Pletnev, 2015)

$$\mathcal{G}^{(1,1)}(1|2) = \frac{(D_1^+)^4(D_2^+)^4}{\widehat{\Box}_1}\frac{\delta^{14}(z_1 - z_2)}{(u_1^+ u_2^+)^3}\,. \tag{16}$$

The result for the divergent contribution is

$$\Gamma^{(1)}_{\rm div} = \frac{1}{6(4\pi)^3\varepsilon}\int d\zeta^{(-4)}(\mathcal{F}^{++}_V + \mathcal{F}^{++}_Q)\big(\mathcal{F}^{++}_V + \mathcal{F}^{++}_Q + 12if^2\widetilde{Q}^+Q^+\big), \tag{17}$$

where we defined the analytic superfield $\mathcal{F}^{++} = D^+_a\mathcal{W}^{+a}$, and then separated both $\mathcal{F}^{++}$ and $\mathcal{W}^{+a}$ in two independent parts

$$\mathcal{W}^{+a} = \mathcal{W}^{+a}_V + \mathcal{W}^{+a}_Q\,, \quad \mathcal{F}^{++} = \mathcal{F}^{++}_V + \mathcal{F}^{++}_Q\,. \tag{18}$$

One can exclude the superfield $\mathcal{F}_V^{++}$ from (17) on the classical equations of motions for background fields

$$\mathcal{F}_V^{++} - 2if^2\widetilde{Q}^+Q^+ = 0\,, \quad \mathcal{D}^{++}Q^+ = 0\,, \tag{19}$$

Nevertheless, the one-loop divergences do not vanish in case of the general self-interaction $L^{(+4)}$.
Consider the example $L^{(+4)} = \frac{1}{2}(\tilde{Q}^+Q^+)^2$. Then $\Psi^{++} = -i\tilde{Q}^+Q^+$.
The divergent part

$$\begin{aligned} \Gamma^{(1)}_{\rm div}[Q^+] \quad &= \quad \frac{1}{6(4\pi)^3\varepsilon}\int d^{14}z\,du\Big(\tilde{Q}^-Q^-\partial^2(\tilde{Q}^+Q^+) + 16f^2\tilde{Q}^-Q^-\tilde{Q}^+Q^+\Big) \\ &\quad -\frac{14f^4}{3(4\pi)^3\varepsilon}\int d\zeta^{(-4)}(\tilde{Q}^+Q^+)^2\,, \end{aligned} \tag{20}$$

it does not vanishes even if we suppose the slowly varying background hypermultiplet, $\partial Q^+ \approx 0$.

Summary

1. We studied the divergent contributions to the one-loop effective action in the model of interacting gauge multiplet and hypermultiplet with arbitrary self-interaction potential function in six dimensions.
2. Using the background field method and proper-time technique in the six-dimensional $\mathcal{N} = (1, 0)$ harmonic superspace, we derived the one-loop contribution to the effective action in the theory under consideration.
3. We calculated the divergent part of one loop effective action in case of the potential $L^{(+4)} = \frac{1}{2}(\tilde{Q}^{+}Q^{+})^{2}$.

Thank you for your attention!