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**GRAVITATIONAL
CHIRAL ANOMALY FOR
FIELDS WITH SPIN
3/2 IN THE RARITA-
SCHWINGER-ADLER
THEORY**

Contents

PART 1

INTRODUCTION

GRAVITY IN FLAT SPACETIME: CHESHIRE CAT GRIN

“Well! I've often seen a cat without a grin,' thought Alice 'but a grin without a cat! It's the most curious thing i ever saw in my life!”

— Lewis Carroll, Alice in Wonderland



GRAVITY IN FLAT SPACETIME: CHESHIRE CAT GRIN

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— Lewis Carroll, Alice in Wonderland



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PART 2

**HYDRODYNAMICS
FROM
THE GRAVITATIONAL
CHIRAL ANOMALY**

DECOMPOSITION OF THE TENSORS

The classification of hydrodynamic effects in curved space-time needs the **expansion of the tensors** into components defined in the rest frame $u_\mu = (1, 0, 0, 0)$:

Thermal vorticity tensor

[M. Buzzegoli, E. Grossi, F. Becattini, JHEP, 10:091, 2017]

$$\varpi_{\mu\nu} = -\frac{1}{2}(\nabla_\mu\beta_\nu - \nabla_\nu\beta_\mu)$$

acceleration: $\alpha_\mu = \varpi_{\mu\nu}u^\nu$

vorticity: $w_\mu = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}u^\nu\varpi^{\alpha\beta}$

Riemann tensor

$$R_{\mu\nu\alpha\beta}$$



In [L.D. Landau, E.M. Lifschits, **Course of Theoretical Physics, Vol. 2**] it is shown that it can be decomposed into 3 three-dimensional tensors:

$$A_{ik}, B_{ik}, C_{ik}$$



Covariant generalization

$$A_{\mu\nu} = u^\alpha u^\beta R_{\alpha\mu\beta\nu}$$

$$B_{\mu\nu} = \frac{1}{2}\epsilon_{\alpha\mu\eta\rho}u^\alpha u^\beta R_{\beta\nu}{}^{\eta\rho}$$

$$C_{\mu\nu} = \frac{1}{4}\epsilon_{\alpha\mu\eta\rho}\epsilon_{\beta\nu\lambda\gamma}u^\alpha u^\beta R^{\eta\rho\lambda\gamma}$$

- Transform to A_{ik}, B_{ik}, C_{ik} in the rest frame $u_\mu = (1, 0, 0, 0)$.
- Have similar properties. In particular, we apply the condition (gravity is an **external** field):

$$R_{\mu\nu} = 0 \quad \Rightarrow \quad A_{\mu\nu} = -C_{\mu\nu}, \quad A^\mu{}_\mu = 0, \quad B_{\mu\nu} = B_{\nu\mu} \quad \Rightarrow \quad 10 \text{ components}$$

GRADIENT EXPANSION IN THE CURVED SPACETIME

The gravitational chiral anomaly has the **4th order** in gradients - it is to be related to the **3rd order** terms in gradient expansion of the axial current.

Hydrodynamic expansion of the axial current up to the **3rd order** in gradients:

$$j_{\mu}^{A(3)} = \xi_1(T)w^2w_{\mu} + \xi_2(T)\alpha^2w_{\mu} + \xi_3(T)(\alpha w)w_{\mu} \\ + \xi_4(T)A_{\mu\nu}w^{\nu} + \xi_5(T)B_{\mu\nu}a^{\nu}$$

Diagram annotations:

- A grey arrow points from the text "arbitrary coefficients" to the $\xi_i(T)$ terms.
- Red arrows point from the text "Survive in flat spacetime" to the w^2w_{μ} , α^2w_{μ} , and $(\alpha w)w_{\mu}$ terms.
- A blue arrow points from the text "'gravitational' currents" to the $A_{\mu\nu}w^{\nu}$ and $B_{\mu\nu}a^{\nu}$ terms.



Substitute to the **anomaly**: $\nabla_{\mu}j_A^{\mu} = \mathcal{N}\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}^{\lambda\rho}$

Global thermodynamic equilibrium [F. Becattini, Acta Phys. Polon. B, 47:1819, 2016.]

$$\nabla_{\mu}\beta_{\nu} + \nabla_{\nu}\beta_{\mu} = 0$$

in particular:

$$\nabla_{\mu}T = T^2\alpha_{\mu} \quad \text{Luttinger relation}$$

$$\nabla_{\mu}w_{\nu} = T(-g_{\mu\nu}(w\alpha) + \alpha_{\mu}w_{\nu}) - T^{-1}B_{\nu\mu}$$

GRAVITATIONAL ANOMALY INDUCED CURRENT

Let's move on to the limit of **flat space-time**. Despite the absence of a gravitational field, there **remains a contribution** to the axial current induced by the gravitational chiral anomaly:

$$\left\{ \begin{array}{l} j_{\mu}^A = \lambda_1(\omega_{\nu}\omega^{\nu})\omega_{\mu} + \lambda_2(a_{\nu}a^{\nu})\omega_{\mu} \quad \Leftarrow \quad R_{\mu\nu\alpha\beta} = 0 \\ \frac{\lambda_1 - \lambda_2}{32} = \mathcal{N} \quad \Leftarrow \quad \nabla_{\mu}j_A^{\mu} = \mathcal{N}\epsilon^{\mu\nu\alpha\beta}R_{\mu\nu\lambda\rho}R_{\alpha\beta}^{\lambda\rho} \end{array} \right.$$

- A new type of anomalous transport - the **Kinematical Vortical Effect (KVE)**.
- Does not explicitly depend on temperature and density \rightarrow determined only by the **kinematics** of the flow.

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PART 3

**VERIFICATION:
SPIN 1/2**

TRANSPORT COEFFICIENTS AND ANOMALY:

SPIN 1/2

- In [GP, O.V. Teryaev, and V.I. Zakharov, JHEP, 02:146, 2019], [V. E. Ambrus, JHEP, 08:016, 2020], [A. Palermo, et al. JHEP 10 (2021) 077] and for ω^3 in [A. Vilenkin, Phys. Rev., D20:1807-1812, 1979] the following expression was obtained:

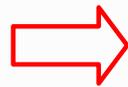
$$j_{\mu}^A = \left(\frac{T^2}{6} + \frac{\mu^2}{2\pi^2} - \overbrace{\left(\frac{\omega^2}{24\pi^2} - \frac{a^2}{8\pi^2} \right)}^{\text{KVE}} \right) \omega_{\mu}$$

- Comparing it with the well-known anomaly [L. Alvarez-Gaume, E. Witten, Nucl. Phys., B234:269, 1984]:

$$\nabla_{\mu} j_{A}^{\mu} = \frac{1}{384\pi^2 \sqrt{-g}} \varepsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}$$

We see that the formula is **fulfilled**:

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$



$$\left(-\frac{1}{24\pi^2} + \frac{1}{8\pi^2} \right) / 32 = \frac{1}{384\pi^2}$$

Correspondence between gravity and hydrodynamics is shown!

- Keeping also the **gravitational** currents, we obtain:

$$j_{\mu}^{A(3)} = \left(-\frac{\omega^2}{24\pi^2} - \frac{a^2}{8\pi^2} \right) \omega_{\mu} + \frac{1}{12\pi^2} B_{\mu\nu} a^{\nu}$$

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PART 4

VERIFICATION:

SPIN 3/2

RARITA-SCHWINGER-ADLER MODEL OF SPIN 3/2

The **Rarita-Schwinger theory** - well-known theory of spin 3/2.
But this theory has a number of **pathologies**.

Generalized Hamiltonian dynamics: **Dirac bracket** instead of **Poisson bracket**

$$[F(\vec{x}), G(\vec{y})]_D = [F(\vec{x}), G(\vec{y})] - \int d^3w d^3z [F(\vec{x}), \chi^\dagger(\vec{w})] M^{-1}(\vec{w}, \vec{z}) [\chi(\vec{z}), G(\vec{y})]$$
$$M(\vec{x}, \vec{y}) = [\chi(\vec{x}), \chi^\dagger(\vec{y})]$$

There is **singularity** in a Dirac bracket in weak gauge field limit for RS-theory!

Doesn't allow to construct perturbation theory!

Solved in [\[Stephen L. Adler. Phys. Rev. D, 97\(4\):045014, 2018\]](#) by introducing of interaction with additional spin 1/2 field:

$$S = \int d^4x \left(-\varepsilon^{\lambda\rho\mu\nu} \bar{\psi}_\lambda \gamma_5 \gamma_\mu \partial_\nu \psi_\rho + i\bar{\lambda} \gamma^\mu \partial_\mu \lambda - im\bar{\lambda} \gamma^\mu \psi_\mu + im\bar{\psi}_\mu \gamma^\mu \lambda \right)$$

GRAVITATIONAL CHIRAL ANOMALY: METHOD OF CONFORMAL THREE-POINT FUNCTIONS

For a conformally symmetric theory, if

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu j_V^\mu = 0, \quad \partial_\mu j_A^\mu = 0, \\ T_\mu^\mu = 0, \quad T_{\mu\nu} = T_{\nu\mu}.$$

It is proven in [J. Erdmenger, Nucl. Phys. B, 562:315–329, 1999], that the three-point function $\langle T\hat{T}_{\mu\nu}(x)\hat{T}_{\sigma\rho}(y)\hat{j}_\omega^A(z)\rangle_c$ has the **universal form**:

$$\langle T\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}_A^\omega(z)\rangle_c = \frac{1}{(x-z)^8(y-z)^8} \\ \times \mathcal{I}_T^{\mu\nu,\mu'\nu'}(x-z)\mathcal{I}_T^{\sigma\rho,\sigma'\rho'}(y-z)t_{\mu'\nu'\sigma'\rho'}^{TTA\omega}(Z)$$

where the notations are introduced:

“6” - consequence of $T_\mu^\mu = 0$

$$\mathcal{I}_{\mu\nu,\sigma\rho}^T(x) = \mathcal{E}_{\mu\nu,\alpha\beta}^T I_\sigma^\alpha(x) I_\rho^\beta(x),$$

$$\mathcal{E}_{\mu\nu,\alpha\beta}^T = \frac{1}{2}(\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}) - \frac{1}{4}\eta_{\mu\nu}\eta_{\alpha\beta},$$

$$t_{\mu\nu\sigma\rho\omega}^{TTA}(\mathcal{Z}) = \frac{\mathcal{A}}{\mathcal{Z}^6} (\mathcal{E}_{\mu\nu,\eta}^T \mathcal{E}_{\sigma\rho,\kappa\varepsilon}^T \varepsilon_\omega^{\eta\kappa\lambda} \mathcal{Z}_\lambda \\ - 6 \mathcal{E}_{\mu\nu,\eta\gamma}^T \mathcal{E}_{\sigma\rho,\kappa\delta}^T \varepsilon_\omega^{\eta\kappa\lambda} \mathcal{Z}^\gamma \mathcal{Z}^\delta \mathcal{Z}_\lambda \mathcal{Z}^{-2})$$

GRAVITATIONAL CHIRAL ANOMALY: METHOD OF CONFORMAL THREE-POINT FUNCTIONS

On the other hand, there is a **gravitational chiral anomaly**:

$$\langle \nabla_{\mu} j_5^{\mu} \rangle = a \frac{\pi^4}{384 \sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

It has been proven that:

$$\mathcal{A} = a$$

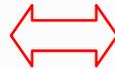
- **The anomaly can be determined by calculating the three-point correlator!**
- The anomaly can be calculated from the correlator in **flat space-time!**
- **No need** to explicitly find a one-loop three-point momentum **divergent** graphs: everything is done in the x-space!

GRAVITATIONAL ANOMALY IN RSA THEORY:

RESULT

Summing 9 correlators, we will obtain:

$$\langle T \hat{T}^{\mu\nu}(x) \hat{T}^{\sigma\rho}(y) \hat{j}_A^\omega(z) \rangle_c = -19 \left(4\pi^6 (x-y)^5 \times (x-z)^3 (y-z)^3 \right)^{-1} e_\vartheta \left(\eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} + \eta^{\mu\rho} \varepsilon^{\sigma\vartheta\nu\omega} + \eta^{\nu\sigma} \varepsilon^{\vartheta\mu\rho\omega} + \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 6e^2 (e^\nu e^\rho \varepsilon^{\sigma\vartheta\mu\omega} + e^\mu e^\rho \varepsilon^{\sigma\vartheta\nu\omega} + e^\sigma e^\nu \varepsilon^{\vartheta\mu\rho\omega} + e^\sigma e^\mu \varepsilon^{\vartheta\nu\rho\omega}) \right)$$



Matches the form we want!

$$\langle T \hat{T}^{\mu\nu}(x) \hat{T}^{\sigma\rho}(y) \hat{j}_A^\omega(z) \rangle = \mathcal{A} \left(4(x-y)^5 \times (x-z)^3 (y-z)^3 \right)^{-1} e_\vartheta \left(\eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} + \eta^{\mu\rho} \varepsilon^{\sigma\vartheta\nu\omega} + \eta^{\nu\sigma} \varepsilon^{\vartheta\mu\rho\omega} + \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 6e^2 (e^\nu e^\rho \varepsilon^{\sigma\vartheta\mu\omega} + e^\mu e^\rho \varepsilon^{\sigma\vartheta\nu\omega} + e^\sigma e^\nu \varepsilon^{\vartheta\mu\rho\omega} + e^\sigma e^\mu \varepsilon^{\vartheta\nu\rho\omega}) \right)$$

(points on the same axis)

We can determine the factor in the anomaly:

$$\mathcal{A}_{RSA} = -19 \mathcal{A}_{s=1/2} = -\frac{19}{\pi^6}$$

$$\langle \nabla_\mu \hat{j}_A^\mu \rangle_{RSA} = \frac{-19}{384\pi^2 \sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

-19 times different from the anomaly for spin 1/2

TRANSPORT COEFFICIENTS: DENSITY OPERATOR

Zubarev **density operator** as a basis for the description of the **vorticity** effects:

$$\hat{\rho} = \frac{1}{Z} \exp \left\{ -\beta_\mu(x) \hat{P}^\mu + \frac{1}{2} \varpi_{\mu\nu} \hat{J}_x^{\mu\nu} + \zeta \hat{Q} \right\} \quad [\text{M. Buzzegoli, E. Grossi, and F. Becattini. JHEP07, 119 (2018).}]$$

Perturbation theory in vorticity gives **KVE**:

$$\langle \hat{j}_5^\lambda(x) \rangle = \dots + \frac{\varpi_{\mu\nu} \varpi_{\rho\sigma} \varpi_{\alpha\beta}}{48 |\beta|^3} \int_0^{|\beta|} d\tau_1 d\tau_2 d\tau_3 \langle T_\tau \hat{J}_{-i\tau_1 u}^{\mu\nu} \hat{J}_{-i\tau_2 u}^{\rho\sigma} \hat{J}_{-i\tau_3 u}^{\alpha\beta} \hat{j}_5^\lambda(0) \rangle_{\beta(x),c}$$

Let's again divide the operators depending on the set of fields:

$$\hat{T}^{\mu\nu} = \hat{T}_{\bar{\psi}\psi}^{\mu\nu} + \hat{T}_{\bar{\lambda}\lambda}^{\mu\nu} + \hat{T}_{\bar{\psi}\lambda}^{\mu\nu} + \hat{T}_{\bar{\lambda}\psi}^{\mu\nu}$$

Initially 5^8 correlators \rightarrow only $\sim 90\,000$ are nonzero

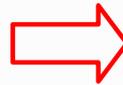
Typical **correlator** to be found (4-point):

$$C^{\alpha_1\alpha_2|\alpha_3\alpha_4|\alpha_5\alpha_6|\lambda|ijk} = \frac{1}{|\beta|^3} \int d\tau_x d\tau_y d\tau_z d^3x d^3y d^3z \times x^i y^j z^k \langle T_\tau \hat{T}^{\alpha_1\alpha_2}(X) \hat{T}^{\alpha_3\alpha_4}(Y) \hat{T}^{\alpha_5\alpha_6}(Z) \hat{j}_A^\lambda(0) \rangle_{\beta(x),c}$$

TRANSPORT COEFFICIENTS AND ANOMALY: SPIN 3/2 INTERACTING WITH SPIN 1/2

The obtained formula for **cubic gradients** (KVE):

$$j_{\mu}^{A(3)} = \left(-\frac{53}{24\pi^2}\omega^2 - \frac{5}{8\pi^2}a^2 \right) \omega_{\mu}$$



Gravitational chiral **anomaly**:

$$\nabla_{\mu} j_A^{\mu} = \frac{-19}{384\pi^2 \sqrt{-g}} \varepsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}$$

$$\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}$$



Direct **verification**:

$$\left(-\frac{53}{24\pi^2} + \frac{5}{8\pi^2} \right) / 32 = -\frac{19}{384\pi^2}$$

Coincidence of hydrodynamics and gravitational anomaly!

- For the RSA theory, the relationship between the transport coefficients in a **vortical accelerated fluid** and the **gravitational** chiral **anomaly** is shown: the factor **-19** from the anomaly is reproduced.
- Verification of the obtained formula in a very **nontrivial** case with higher spins and interaction.

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PART 5

CONCLUSION

CONCLUSION

- The relationship between the hydrodynamic current in the third order of gradient expansion $(\omega_\nu \omega^\nu) \omega_\mu$ and $(a_\nu a^\nu) w_\mu$, the Kinematical Vortical Effect (**KVE**), and the **gravitational chiral anomaly** has been proven:
 - The axial current in a flat space-time in a vortical and accelerated fluid turns out to be associated with a quantum violation of current conservation in a curved space-time.
- The obtained formula has been **verified** directly for **spin 1/2**.
- The obtained formula has been **verified** for **spin 3/2** using the **RSA** theory:
 - Cubic transport coefficients were derived using the statistical density operator expansion $-53/(24\pi^2)\omega^3$ and $-5/(8\pi^2)a^2\omega$.
 - The gravitational chiral anomaly was found by the method of conformal three-point functions: the factor in the anomaly is $-19/(384\pi^2)$.
 - Correspondence between the KVE and the gravitational chiral anomaly is directly shown $[-53/(24\pi^2) + 5/(8\pi^2)] = -19/(384\pi^2)$.

ADDITIONAL SLIDES

$$\begin{aligned} R_{\mu\nu\alpha\beta} &= u_\mu u_\alpha A_{\nu\beta} + u_\nu u_\beta A_{\mu\alpha} - u_\nu u_\alpha A_{\mu\beta} - u_\mu u_\beta A_{\nu\alpha} \\ &\quad + \epsilon_{\mu\nu\lambda\rho} u^\rho (u_\alpha B^\lambda_\beta - u_\beta B^\lambda_\alpha) \\ &\quad + \epsilon_{\alpha\beta\lambda\rho} u^\rho (u_\mu B^\lambda_\nu - u_\nu B^\lambda_\mu) \\ &\quad + \epsilon_{\mu\nu\lambda\rho} \epsilon_{\alpha\beta\eta\sigma} u^\rho u^\sigma C^{\lambda\eta}. \end{aligned}$$

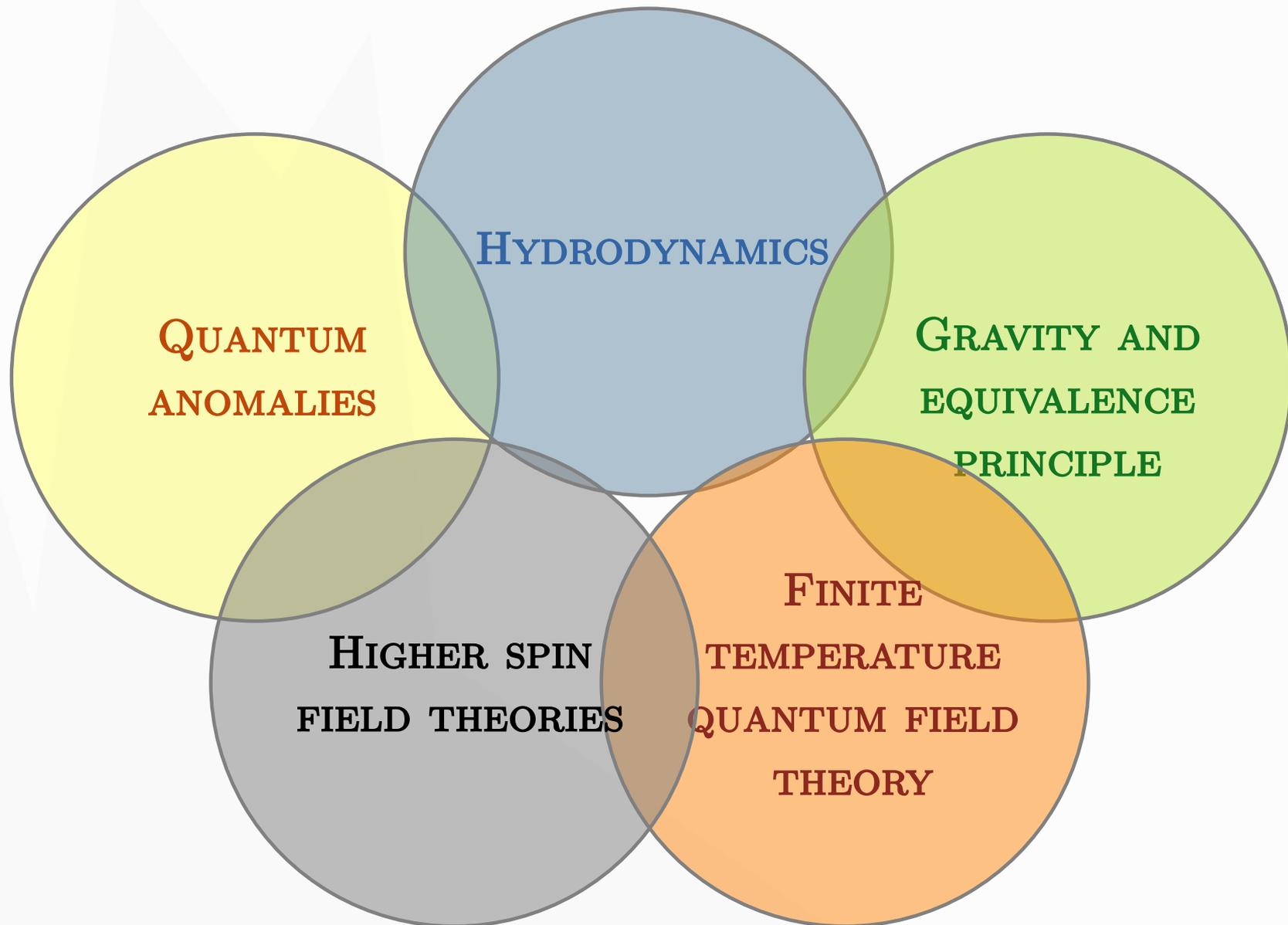
$$\langle T^{\mu\nu} \rangle = -\frac{2}{\sqrt{-g}} \frac{\delta \mathcal{W}}{\delta g_{\mu\nu}}$$

$$\begin{aligned} \langle T \hat{T}^{\mu\nu}(x) \hat{T}^{\sigma\rho}(y) \hat{j}_A^\omega(z) \rangle_c &= \frac{4}{\sqrt{-g(x)} \sqrt{-g(y)} \sqrt{-g(z)}} \\ &\times \frac{\delta}{\delta g_{\mu\nu}(x)} \frac{\delta}{\delta g_{\sigma\rho}(y)} \left(\sqrt{-g(z)} \langle \hat{j}_A^\omega(z) \rangle \right), \end{aligned}$$

CONTENTS

- Introduction
- **Hydrodynamics** from the **gravitational** chiral anomaly:
 - **Kinematical** Vortical Effect (**KVE**)
- Verification: spin $1/2$
- Verification: **spin $3/2$**
 - Rarita-Schwinger-Adler (RSA) theory
 - Gravitational chiral anomaly
 - **KVE vs anomaly**

WHAT WILL BE DISCUSSED?

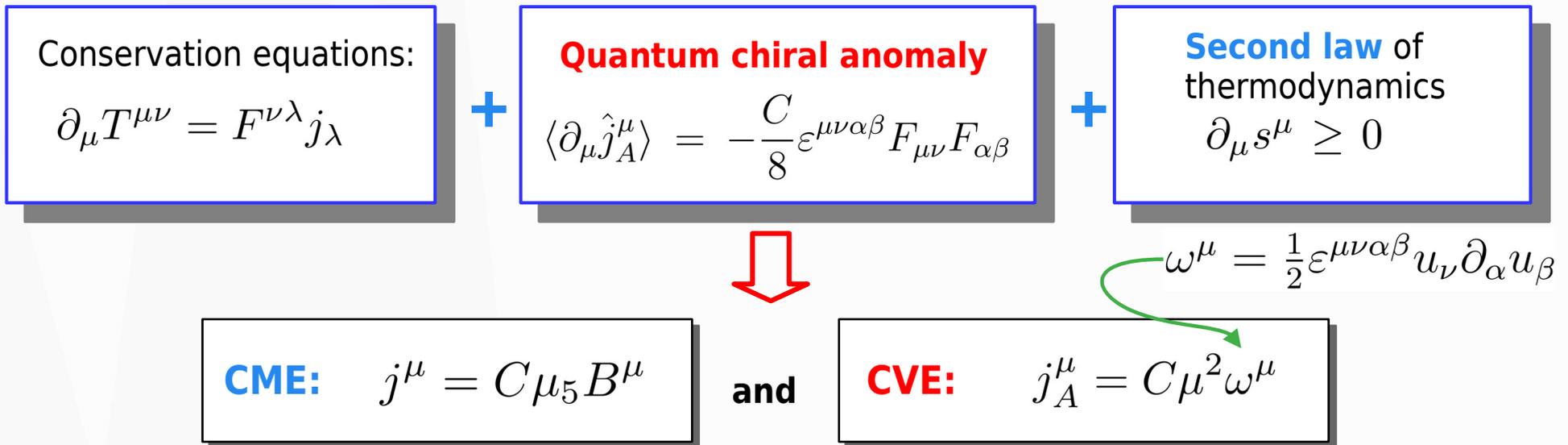


MANIFESTATION OF THE GAUGE CHIRAL ANOMALY

Accounting for quantum field effects in hydrodynamic equations

[D.T. Son, P. Surowka, PRL, 103 (2009) 191601]:

- Generalization of [L.D. Landau, E.M. Lifshits, Hydrodynamics, vol. VI. Nauka, 1986]



- In [Shi-Zheng Yang, et al. Symmetry, 2022] and [M. Buzzegoli, Lect.Notes Phys., 2021] it was shown that in the global equilibrium [F. Becattini. Acta Phys. Polon. B, 2016]:

$$\partial_\mu \beta_\nu + \partial_\nu \beta_\mu = 0 \quad , \quad \partial_\mu \frac{\mu}{T} = -\frac{E_\mu}{T}$$

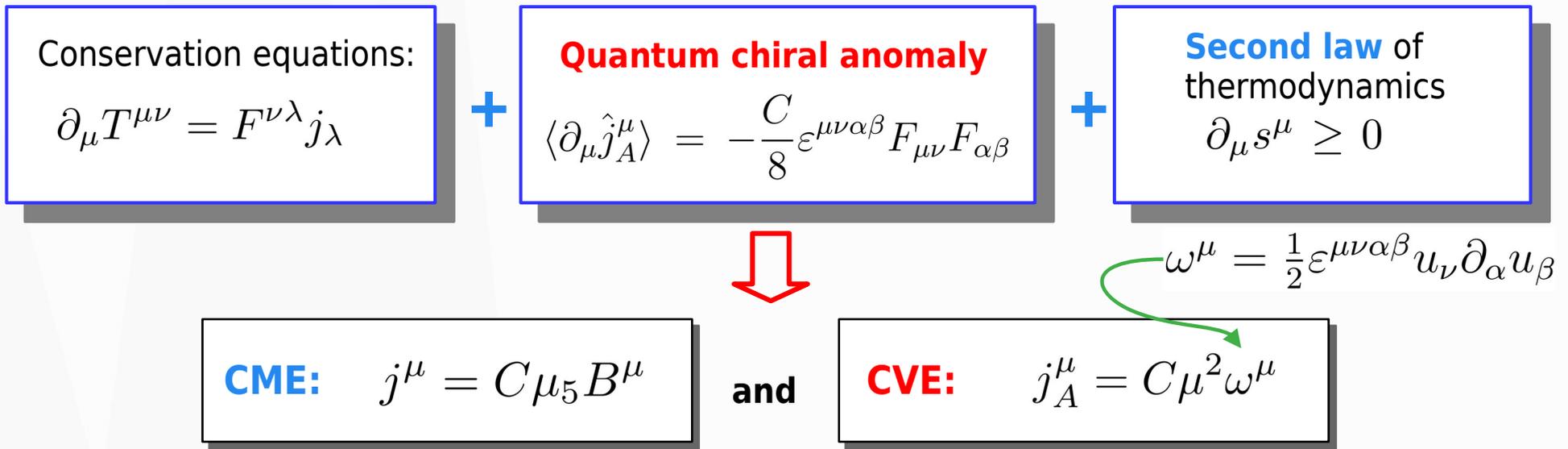
for a non-dissipative fluid, only the **current conservation** condition can be applied instead of the entropy non-increase law.

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MODERN DEVELOPMENT AND THE PROBLEM

A whole set of similar effects has been found at the intersection of quantum field theory and hydrodynamics [M. N. Chernodub et al. 2110.05471].

Experimental search is in progress:

- CME not yet found in ion collisions at RHIC [D.E. Kharzeev et al. 2205.00120].
- Condensed matter copies of the effects are found in semimetals [Qiang Li et al. Nature Phys. 12 (2016)].

What about the **gravitational chiral anomaly**?

- A connection between the thermal chiral effect $\mathbf{j}_A \sim T^2 \boldsymbol{\Omega}$ and the gravitational anomaly is predicted [K. Landsteiner et al. Phys. Rev. Lett., 107:021601, 2011].

- However, the anomaly grows **rapidly** with **spin** [M. J. Duff, 1982]:
$$\langle \nabla_\mu \hat{j}_A^\mu \rangle_S = \frac{(S - 2S^3)}{96\pi^2 \sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

- For the thermal CVE - the linear dependence on spin [GP, O.V. Teryaev, V.I. Zakharov, Phys. Rev. D, 102(12):121702(R), 2020].

How does the **gravitational** chiral anomaly manifest itself in **hydrodynamics**? Can the analysis [D.T. Son and P. Surowka, PRL, 2009] be **generalized** to the case of **curved** space-time?

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CONSERVATION EQUATION: SYSTEM OF EQUATIONS

Independent pseudoscalars

$$\begin{aligned}
 \nabla_{\mu} j_{A(3)}^{\mu} &= (\alpha w) w^2 (-3T \xi_1 + T^2 \xi_1' + 2T \xi_3) \\
 &+ (\alpha w) \alpha^2 (-3T \xi_2 + T^2 \xi_2' - T \xi_3 + T^2 \xi_3') \\
 &+ A_{\mu\nu} \alpha^{\mu} w^{\nu} (T^2 \xi_4' + 3T \xi_5 + 2T^{-1} \xi_2 + T^{-1} \xi_3) \\
 &+ B_{\mu\nu} w^{\mu} w^{\nu} (-2T^{-1} \xi_1 - 3T \xi_4 - T \xi_5) \\
 &+ B_{\mu\nu} \alpha^{\mu} \alpha^{\nu} (T^2 \xi_5' - T \xi_5 - T^{-1} \xi_3) \\
 &+ A_{\mu\nu} B^{\mu\nu} (-T^{-1} \xi_4 + T^{-1} \xi_5) \\
 &= 32 \mathcal{N} A_{\mu\nu} B^{\mu\nu} \cdot \text{Gravitational chiral anomaly}
 \end{aligned}$$

The coefficient in front of each pseudoscalar is to be **zero** → the **system of equations** for the unknown **coefficients** $\xi_n(T)$.

SOLUTION: KINEMATICAL VORTICAL EFFECT

The system of **equations** has the form:

The **solution** looks like:

$$\left\{ \begin{array}{l} -3T\xi_1 + T^2\xi'_1 + 2T\xi_3 = 0, \\ -3T\xi_2 + T^2\xi'_2 - T\xi_3 + T^2\xi'_3 = 0, \\ T^2\xi'_4 + 3T\xi_5 + 2T^{-1}\xi_2 + T^{-1}\xi_3 = 0, \\ -2T^{-1}\xi_1 - 3T\xi_4 - T\xi_5 = 0, \\ T^2\xi'_5 - T\xi_5 - T^{-1}\xi_3 = 0, \\ -T^{-1}\xi_4 + T^{-1}\xi_5 - 32\mathcal{N} = 0. \end{array} \right. \Rightarrow \left\{ \begin{array}{l} \lambda_3 = 0 \\ \lambda_4 = -8\mathcal{N} - \frac{\lambda_1}{2} \\ \lambda_5 = 24\mathcal{N} - \frac{\lambda_1}{2} \\ \boxed{\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N}} \end{array} \right.$$

(if there are no **dimensional** parameters other than temperature T)

$$\xi_1 = \lambda_1 T^3, \xi_2 = \lambda_2 T^3 \dots$$

- $\lambda_3 = 0 \rightarrow$ conservation of current in flat space-time [GP, O.V. Teryaev, V.I. Zakharov, [Phys. Rev. D, 105\(4\):L041701, 2022](#)].
- $\frac{\lambda_1 - \lambda_2}{32} = \mathcal{N} \rightarrow$ fixes the relationship of the axial current in an **accelerated** and **vortical** flow with a **gravitational** chiral quantum anomaly.

KVE AND UNRUH EFFECT

- It is possible to distinguish **conserved** and **anomalous** parts of the current:

$$j_{\mu}^A = j_{\mu(\text{conserv})}^A + j_{\mu(\text{anom})}^A$$

$$\text{Thermal vorticity tensor squared } \omega^2 - a^2 = -\frac{T^2}{2} \varpi_{\mu\nu} \varpi^{\mu\nu}$$

$$j_{\mu(\text{anom})}^A = 16\mathcal{N} \left\{ (\omega^2 - a^2) \omega_{\mu} - A_{\mu\nu} \omega^{\nu} + B_{\mu\nu} a^{\nu} \right\} \Rightarrow \nabla^{\mu} j_{\mu(\text{anom})}^A = \mathcal{N} \epsilon^{\mu\nu\alpha\beta} R_{\mu\nu\lambda\rho} R_{\alpha\beta}{}^{\lambda\rho}$$

$$j_{\mu(\text{conserv})}^A = \frac{\lambda_1 + \lambda_2}{2} \left\{ (\omega^2 + a^2) \omega_{\mu} - \frac{1}{2} A_{\mu\nu} \omega^{\nu} - \frac{1}{2} B_{\mu\nu} a^{\nu} \right\} \Rightarrow \nabla^{\mu} j_{\mu(\text{conserv})}^A = 0$$

- Consider the term with acceleration from the anomalous part of the current:

$$j_{\mu(\text{anom})}^A = -16\mathcal{N} a^2 \omega_{\mu}$$

- Unruh effect** [W.G. Unruh, 1976] - in an accelerated frame there is a thermal bath of particles with the **Unruh temperature**:

$$T_U = |a|/(2\pi)$$

Substitute $|a| \rightarrow 2\pi T_U$:

$$j_{\mu(\text{anom})}^A = \frac{T_U^2}{6} \omega_{\mu} \quad \text{for spin } 1/2 \rightarrow \text{standard CVE}$$

$$j_{\mu(\text{anom})}^A = 64\pi^2 \mathcal{N} T_U^2 \omega_{\mu}$$

thermal CVE current is **proportional** to the **anomaly**!

- Match with [K. Landsteiner, et al. PRL, 2011] and [M. Stone, J. Kim. PRD, 2018], where **thermal CVE** $\mathbf{j}_A \sim T^2 \boldsymbol{\Omega}$ is associated with the **gravitational chiral anomaly**!

GRAVITATIONAL CHIRAL ANOMALY: METHOD OF CONFORMAL THREE-POINT FUNCTIONS

We will consider the case of points on one 4-axis:

$$x_\mu = x e_\mu, \quad y_\mu = y e_\mu, \quad z_\mu = z e_\mu$$

Then the correlator should look like:

$$\begin{aligned} \langle T \hat{T}^{\mu\nu}(x) \hat{T}^{\sigma\rho}(y) \hat{j}_A^\omega(z) \rangle = & \mathcal{A} \left(4(x-y)^5 \right. \\ & \times (x-z)^3 (y-z)^3 \left. \right)^{-1} e_\vartheta \left(\eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} + \eta^{\mu\rho} \varepsilon^{\sigma\vartheta\nu\omega} \right. \\ & + \eta^{\nu\sigma} \varepsilon^{\vartheta\mu\rho\omega} + \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 6e^2 (e^\nu e^\rho \varepsilon^{\sigma\vartheta\mu\omega} \\ & \left. + e^\mu e^\rho \varepsilon^{\sigma\vartheta\nu\omega} + e^\sigma e^\nu \varepsilon^{\vartheta\mu\rho\omega} + e^\sigma e^\mu \varepsilon^{\vartheta\nu\rho\omega}) \right) \end{aligned}$$

GRAVITATIONAL ANOMALY IN RSA THEORY: DIAGRAMS

Let's decompose all the operators depending on the set of the fields:

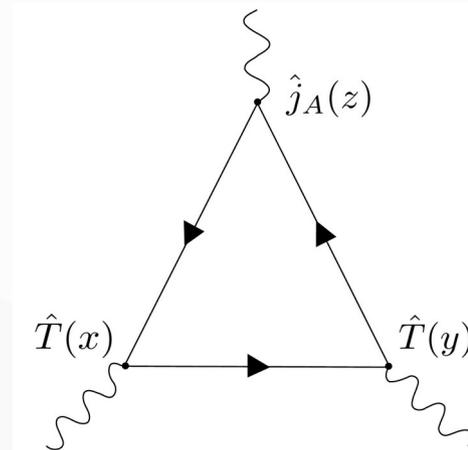
$$\hat{T}^{\mu\nu} = \hat{T}_{\bar{\psi}\psi}^{\mu\nu} + \hat{T}_{\bar{\lambda}\lambda}^{\mu\nu} + \hat{T}_{\bar{\psi}\lambda}^{\mu\nu} + \hat{T}_{\bar{\lambda}\psi}^{\mu\nu} \quad \hat{j}_A^\mu = \hat{j}_{A\bar{\psi}\psi}^\mu + \hat{j}_{A\bar{\lambda}\lambda}^\mu$$

Then the three-point function decomposes into **32 terms**:

$$\langle T\hat{T}^{\mu\nu}(x)\hat{T}^{\sigma\rho}(y)\hat{j}_A^\omega(z) \rangle_c = \langle \hat{T}_{\bar{\psi}\psi}\hat{T}_{\bar{\psi}\psi}\hat{j}_{\bar{\psi}\psi}^A \rangle + (31 \text{ terms})$$

However, many are equal to **zero** or depend on each other: there are only **4 independent** correlators.

A **typical** diagram
(different SETs in the vertices):



GRAVITATIONAL ANOMALY IN RSA THEORY: CALCULATION DETAILS

As a result, we have for the independent correlators:

$$\langle T \hat{T}_{\bar{\psi}\psi}^{\mu\nu}(x) \hat{T}_{\bar{\psi}\psi}^{\sigma\rho}(y) \hat{j}_{A\bar{\psi}\psi}^{\omega}(z) \rangle_c = \frac{1}{4\pi^6(x-y)^5(x-z)^4(y-z)^4} e_{\vartheta} (28e^2 x^2 e^{\mu} e^{\sigma} \varepsilon^{\vartheta\nu\rho\omega} - \varepsilon^{\sigma\vartheta\nu\omega} (2e^2 e^{\mu} e^{\rho} (-14x^2 + 9z(x+y) + 19xy - 14y^2 - 9z^2) + (26x^2 - 3z(x+y) - 49xy + 26y^2 + 3z^2) \eta^{\mu\rho}) + 2e^2 e^{\nu} (14x^2 - 19xy - 9xz + 14y^2 - 9yz + 9z^2) (e^{\rho} \varepsilon^{\sigma\vartheta\mu\omega} + e^{\sigma} \varepsilon^{\vartheta\mu\rho\omega}) - 38e^2 xy e^{\mu} e^{\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 18e^2 xz e^{\mu} e^{\sigma} \varepsilon^{\vartheta\nu\rho\omega} + 28e^2 y^2 e^{\mu} e^{\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 18e^2 yz e^{\mu} e^{\sigma} \varepsilon^{\vartheta\nu\rho\omega} + 18e^2 z^2 e^{\mu} e^{\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 26x^2 \eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} - 26x^2 \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} - (26x^2 - 3z(x+y) - 49xy + 26y^2 + 3z^2) \eta^{\nu\sigma} \varepsilon^{\vartheta\mu\rho\omega} + 49xy \eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} + 49xy \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} + 3xz \eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} + 3xz \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 26y^2 \eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} - 26y^2 \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} + 3yz \eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} + 3yz \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 3z^2 \eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} - 3z^2 \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega}),$$

$$\langle T \hat{T}_{\bar{\psi}\psi}^{\mu\nu}(x) \hat{T}_{\bar{\psi}\lambda}^{\sigma\rho}(y) \hat{j}_{A\bar{\psi}\psi}^{\omega}(z) \rangle_c = \frac{1}{4\pi^6(x-y)^5(x-z)^4(y-z)^4} e_{\vartheta} (4e^2 x^2 e^{\mu} e^{\sigma} \varepsilon^{\vartheta\nu\rho\omega} - \varepsilon^{\sigma\vartheta\nu\omega} (2e^2 e^{\mu} e^{\rho} (-2x^2 + 3z(7x+y) - 17xy + 7y^2 - 12z^2) + (10x^2 + 7xy - 27xz - 13y^2 + 19yz + 4z^2) \eta^{\mu\rho}) + 2e^2 e^{\nu} (2x^2 + 17xy - 21xz - 7y^2 - 3yz + 12z^2) (e^{\rho} \varepsilon^{\sigma\vartheta\mu\omega} + e^{\sigma} \varepsilon^{\vartheta\mu\rho\omega}) + 34e^2 xy e^{\mu} e^{\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 42e^2 xz e^{\mu} e^{\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 14e^2 y^2 e^{\mu} e^{\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 6e^2 yz e^{\mu} e^{\sigma} \varepsilon^{\vartheta\nu\rho\omega} + 24e^2 z^2 e^{\mu} e^{\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 10x^2 \eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} - 10x^2 \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} - (10x^2 + 7xy - 27xz - 13y^2 + 19yz + 4z^2) \eta^{\nu\sigma} \varepsilon^{\vartheta\mu\rho\omega} - 7xy \eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} - 7xy \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} + 27xz \eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} + 27xz \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} + 13y^2 \eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} + 13y^2 \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 19yz \eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} - 19yz \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 4z^2 \eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} - 4z^2 \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega}),$$

$$\langle T \hat{T}_{\bar{\psi}\lambda}^{\mu\nu}(x) \hat{T}_{\bar{\psi}\lambda}^{\sigma\rho}(y) \hat{j}_{A\bar{\psi}\psi}^{\omega}(z) \rangle_c = \frac{4e^2 e_{\vartheta} (e^{\nu} (e^{\rho} \varepsilon^{\sigma\vartheta\mu\omega} + e^{\sigma} \varepsilon^{\vartheta\mu\rho\omega}) + e^{\mu} (e^{\rho} \varepsilon^{\sigma\vartheta\nu\omega} + e^{\sigma} \varepsilon^{\vartheta\nu\rho\omega}))}{\pi^6(x-y)^3(x-z)^4(y-z)^4}$$

$$\langle T \hat{T}_{\bar{\psi}\lambda}^{\mu\nu}(x) \hat{T}_{\bar{\lambda}\psi}^{\sigma\rho}(y) \hat{j}_{A\bar{\psi}\psi}^{\omega}(z) \rangle_c = \frac{5}{2\pi^6(x-y)^3(x-z)^4(y-z)^4} e_{\vartheta} (-2e^2 e^{\mu} e^{\rho} \varepsilon^{\sigma\vartheta\nu\omega} - 2e^2 e^{\nu} (e^{\rho} \varepsilon^{\sigma\vartheta\mu\omega} + e^{\sigma} \varepsilon^{\vartheta\mu\rho\omega}) - 2e^2 e^{\mu} e^{\sigma} \varepsilon^{\vartheta\nu\rho\omega} + \eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} + \eta^{\mu\rho} \varepsilon^{\sigma\vartheta\nu\omega} + \eta^{\sigma\nu} \varepsilon^{\vartheta\mu\rho\omega} + \eta^{\sigma\mu} \varepsilon^{\vartheta\nu\rho\omega}).$$

Each term **differs** from what we need.

GRAVITATIONAL ANOMALY IN RSA THEORY:

RESULT

- **How to explain the factor -19?**
- How does it **relate** to **previous** calculations?

$$\langle \nabla_{\mu} \hat{j}_A^{\mu} \rangle_{RS} = \frac{-21}{384\pi^2 \sqrt{-g}} \epsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

[M. J. Duff, 1201.0386] [J.J.M. Carrasco, et al. JHEP, 07:029, 2013]

“ghostless” contribution [J.J.M. Carrasco, et al. JHEP, 07:029, 2013]

$$-19 = -20 + 1$$

$$-19 = -21 + 2$$

Contribution
of spin 1/2

Gauge anomaly

$$5 = 4 + 1$$

$$5 = 3 + 2$$

TRANSPORT COEFFICIENTS: CORRELATORS

The transport coefficients are expressed in terms of a combination of the correlators:

$$\xi_1 = -\frac{1}{6} \left(C^{02|02|02|3|111} + C^{02|01|01|3|122} + C^{01|02|01|3|212} \right. \\ \left. + C^{01|01|02|3|221} - C^{01|01|01|3|222} - C^{01|02|02|3|211} \right. \\ \left. - C^{02|01|02|3|121} - C^{02|02|01|3|112} \right),$$

$$\xi_2 = -\frac{1}{6} \left(C^{02|00|00|3|111} + C^{00|02|00|3|111} + C^{00|00|02|3|111} \right. \\ \left. - C^{01|00|00|3|211} - C^{00|01|00|3|121} \right. \\ \left. - C^{00|00|01|3|112} \right).$$

Direct calculation within the finite temperature QFT:

$$\xi_1 = -\frac{T^3}{6} \left(\frac{177}{80\pi^2} + \frac{353}{240\pi^2} + \frac{353}{240\pi^2} + \frac{353}{240\pi^2} \right. \\ \left. + \frac{177}{80\pi^2} + \frac{353}{240\pi^2} + \frac{353}{240\pi^2} + \frac{353}{240\pi^2} \right) = -\frac{53}{24\pi^2} T^3$$

$$\xi_2 = -\frac{T^3}{6} \left(\frac{33}{40\pi^2} + \frac{53}{80\pi^2} + \frac{1}{2\pi^2} + \frac{3}{4\pi^2} \right. \\ \left. + \frac{47}{80\pi^2} + \frac{17}{40\pi^2} \right) = -\frac{5}{8\pi^2} T^3$$

Cubic gradients (**KVE**) in the **RSA** theory:

$$j_\mu^{A(3)} = \left(-\frac{53}{24\pi^2} \omega^2 - \frac{5}{8\pi^2} a^2 \right) \omega_\mu$$

GRAVITY IN FLAT SPACETIME: CHESHIRE CAT GRIN



Method of conformal correlation functions

[GP, O.V. Teryaev, V.I. Zakharov. 2202.02168]

- There are various ways to calculate quantum anomalies.
- **One of the methods:**
 - [J. Erdmenger, H. Osborn. Nucl. Phys. B, 483:431–474, 1997] → Gauge chiral anomaly
 - [J. Erdmenger. Nucl. Phys. B, 562:315–329, 1999] → Gravitational chiral anomaly
- It is shown that three-point functions have a **universal form**. In particular, for the function with the currents:

$$\begin{aligned} \langle T \hat{j}_V^\mu(x) \hat{j}_V^\nu(y) \hat{j}_A^\omega(z) \rangle_c = \\ = -4\mathcal{B} \frac{I_{\mu'}^\mu(x-z) I_{\nu'}^\nu(y-z)}{(x-z)^6 (y-z)^6} \varepsilon^{\mu'\nu'\lambda\omega} \frac{Z_\lambda}{Z^4} \end{aligned}$$

Designations introduced:

$$\begin{aligned} Z_\mu &= \frac{(x-z)_\mu}{(x-z)^2} - \frac{(x-y)_\mu}{(x-y)^2} \\ I_{\mu\nu}(x) &= \eta_{\mu\nu} - 2 \frac{x_\mu x_\nu}{x^2} \end{aligned}$$

It will be so for a conformally symmetric theory, if

$$\begin{aligned} \partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu j_V^\mu = 0, \quad \partial_\mu j_A^\mu = 0, \\ T_\mu^\mu = 0, \quad T_{\mu\nu} = T_{\nu\mu}. \end{aligned}$$

Method of conformal correlation functions: gauge chiral anomaly

- Can this method be used to calculate anomalies in the RSA theory?

$$T_{\mu}^{\mu} = \frac{1}{2} \varepsilon^{\lambda\mu\beta\rho} \left(\bar{\psi}_{\lambda} \gamma_5 \gamma_{\mu} \partial_{\beta} \psi_{\rho} - \partial_{\beta} \bar{\psi}_{\lambda} \gamma_5 \gamma_{\mu} \psi_{\rho} \right) \\ + i \partial_{\eta} \left[(\bar{\psi} \gamma) \psi^{\eta} - \bar{\psi}^{\eta} (\gamma \psi) \right] \\ + \frac{i}{2} \left[\bar{\lambda} (\gamma \partial) \lambda - (\partial \bar{\lambda} \gamma) \lambda \right] \\ + i m \left[(\bar{\psi} \gamma) \lambda - \bar{\lambda} (\gamma \psi) \right].$$

The SET trace is equal to zero just in the extended theory!

$$\text{RS theory : } T_{\mu}^{\mu} = i \partial_{\eta} \left[(\bar{\psi} \gamma) \psi^{\eta} - \bar{\psi}^{\eta} (\gamma \psi) \right] \neq 0,$$

$$\text{RSA theory : } T_{\mu}^{\mu} = 0.$$

[M. N. Chernodub, et al. 2110.05471],
[Yu Nakayama. Phys. Rept., 569:1–93, 2015]

- **We can use the described method for calculating anomalies!**

Method of conformal correlation functions: gauge chiral anomaly

Propagators in momentum representation, for example:

$$\langle T \psi_a^\rho(x) \bar{\psi}_b^\sigma(0) \rangle = \frac{i}{2(2\pi)^4} \int \frac{d^4 p}{p^2} \left(\gamma^\sigma \not{p} \gamma^\rho - 2 \left(\frac{1}{m^2} + \frac{2}{p^2} \right) p^\sigma p^\rho \not{p} \right)_{ab} e^{-ipx}$$

using the formula (*I am grateful to A. Pikelner and A. Bednyakov for the link*):

$$\int \frac{d^D p e^{ipx}}{p^{2(\lambda+1-\alpha)}} = \frac{i 2^{2\alpha} \pi^{\lambda+1} \Gamma(\alpha)}{x^{2\alpha} \Gamma(\lambda+1-\alpha)}$$
$$\lambda = 1 - \varepsilon, \quad D = 2(\lambda + 1),$$

can be translated into a **coordinate representation** (need to find momentum integrals).

Method of conformal correlation functions: gauge chiral anomaly

As a result, we obtain for the **propagators** in the **coordinate representation**:

$$\begin{aligned}\langle T\psi_a^\rho(x)\bar{\psi}_b^\sigma(0)\rangle &= \frac{i}{4\pi^2x^4} \left[\gamma^\sigma \not{x} \gamma^\rho - 2 \left(1 + \frac{4}{m^2x^2} \right) \right. \\ &\times (\eta^{\rho\sigma} \not{x} + \gamma^\rho x^\sigma + \gamma^\sigma x^\rho) + 8 \left(1 + \frac{6}{m^2x^2} \right) \frac{x^\rho x^\sigma \not{x}}{x^2} \left. \right]_{ab} \\ \langle T\lambda_a(x)\bar{\psi}_b^\sigma(0)\rangle &= \frac{i}{2\pi^2mx^4} \left(\gamma^\sigma - \frac{4x^\sigma \not{x}}{x^2} \right)_{ab}, \\ \langle T\psi_a^\rho(x)\bar{\lambda}_b(0)\rangle &= \frac{-i}{2\pi^2mx^4} \left(\gamma^\rho - \frac{4x^\rho \not{x}}{x^2} \right)_{ab}, \\ \langle T\lambda_a(x)\bar{\lambda}_b(0)\rangle &= 0.\end{aligned}$$

Method of conformal correlation functions: gauge chiral anomaly

As a result, we obtain:

$$\langle T \hat{j}^\mu(x) \hat{j}^\nu(y) \hat{j}_A^\omega(z) \rangle_c = \frac{5e^2 e_\vartheta \varepsilon^{\vartheta\mu\nu\omega}}{\pi^6 (x-y)^3 (x-z)^3 (y-z)^3}$$

Comparing with the general form

$$\langle T \hat{j}_V^\mu(x) \hat{j}_V^\nu(y) \hat{j}_A^\omega(z) \rangle = \frac{4\mathcal{B} e^2 e_\vartheta \varepsilon^{\vartheta\mu\nu\omega}}{\pi^6 (x-y)^3 (x-z)^3 (y-z)^3}$$

we obtain an anomaly factor:

$$\langle \partial_\mu \hat{j}_A^\mu \rangle_{RSA} = -\frac{5}{16\pi^2} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

which confirms the result from [S.L. Adler. Phys. Rev. D, 97(4):045014, 2018].

Also checked for arbitrary x, y, z .

Method of conformal correlation functions: gravitational chiral anomaly

We will consider the case of points on one 4-axis:

$$x_\mu = x e_\mu, \quad y_\mu = y e_\mu, \quad z_\mu = z e_\mu$$

Then the correlator should look like:

$$\begin{aligned} \langle T \hat{T}^{\mu\nu}(x) \hat{T}^{\sigma\rho}(y) \hat{j}_A^\omega(z) \rangle = & \mathcal{A} \left(4(x-y)^5 \right. \\ & \times (x-z)^3 (y-z)^3 \left. \right)^{-1} e_\vartheta \left(\eta^{\nu\rho} \varepsilon^{\sigma\vartheta\mu\omega} + \eta^{\mu\rho} \varepsilon^{\sigma\vartheta\nu\omega} \right. \\ & + \eta^{\nu\sigma} \varepsilon^{\vartheta\mu\rho\omega} + \eta^{\mu\sigma} \varepsilon^{\vartheta\nu\rho\omega} - 6e^2 (e^\nu e^\rho \varepsilon^{\sigma\vartheta\mu\omega} \\ & \left. + e^\mu e^\rho \varepsilon^{\sigma\vartheta\nu\omega} + e^\sigma e^\nu \varepsilon^{\vartheta\mu\rho\omega} + e^\sigma e^\mu \varepsilon^{\vartheta\nu\rho\omega}) \right) \end{aligned}$$

Method of conformal correlation functions: gravitational chiral anomaly

$\langle T\lambda_a(x)\bar{\lambda}_b(0)\rangle = 0$ The field λ is **nonpropagating**.

$$\begin{aligned} \langle \hat{T}_{\bar{\psi}\psi} \hat{T}_{\bar{\lambda}\lambda} \hat{j}_{\bar{\lambda}\lambda}^A \rangle &= \langle \hat{T}_{\bar{\psi}\lambda} \hat{T}_{\bar{\psi}\lambda} \hat{j}_{\bar{\lambda}\lambda}^A \rangle = \langle \hat{T}_{\bar{\psi}\lambda} \hat{T}_{\bar{\lambda}\psi} \hat{j}_{\bar{\lambda}\lambda}^A \rangle = \\ &= \langle \hat{T}_{\bar{\psi}\lambda} \hat{T}_{\bar{\lambda}\lambda} \hat{j}_{\bar{\lambda}\lambda}^A \rangle = \langle \hat{T}_{\bar{\lambda}\psi} \hat{T}_{\bar{\psi}\lambda} \hat{j}_{\bar{\lambda}\lambda}^A \rangle = \langle \hat{T}_{\bar{\lambda}\psi} \hat{T}_{\bar{\lambda}\psi} \hat{j}_{\bar{\lambda}\lambda}^A \rangle = \\ &= \langle \hat{T}_{\bar{\lambda}\psi} \hat{T}_{\bar{\lambda}\lambda} \hat{j}_{\bar{\lambda}\lambda}^A \rangle = \langle \hat{T}_{\bar{\lambda}\lambda} \hat{T}_{\bar{\psi}\psi} \hat{j}_{\bar{\lambda}\lambda}^A \rangle = \langle \hat{T}_{\bar{\lambda}\lambda} \hat{T}_{\bar{\psi}\lambda} \hat{j}_{\bar{\lambda}\lambda}^A \rangle = \\ &= \langle \hat{T}_{\bar{\lambda}\lambda} \hat{T}_{\bar{\lambda}\psi} \hat{j}_{\bar{\lambda}\lambda}^A \rangle = \langle \hat{T}_{\bar{\lambda}\lambda} \hat{T}_{\bar{\lambda}\psi} \hat{j}_{\bar{\psi}\psi}^A \rangle = \langle \hat{T}_{\bar{\lambda}\lambda} \hat{T}_{\bar{\lambda}\lambda} \hat{j}_{\bar{\lambda}\lambda}^A \rangle = 0 \end{aligned}$$

- **12** correlators **are zero!**

Method of conformal correlation functions: gravitational chiral anomaly

$$\langle T\lambda_a(x)\bar{\psi}_b^\sigma(0)\rangle = \frac{i}{2\pi^2 m x^4} \left(\gamma^\sigma - \frac{4x^\sigma \not{x}}{x^2} \right)_{ab} \quad \Rightarrow \text{negative powers } m$$

$$\hat{T}_{\bar{\psi}\lambda}^{\mu\nu} = \frac{i}{2} m \left(\bar{\psi}^\mu \gamma^\nu \lambda + \bar{\psi}^\nu \gamma^\mu \lambda \right) \quad \Rightarrow \text{positive powers } m$$

In the limit $m \rightarrow \infty$, **11** more correlators **vanish**:

$$\begin{aligned} & \langle \hat{T}_{\bar{\psi}\psi} \hat{T}_{\bar{\psi}\psi} \hat{j}_{\bar{\lambda}\lambda}^A \rangle, \langle \hat{T}_{\bar{\psi}\psi} \hat{T}_{\bar{\psi}\lambda} \hat{j}_{\bar{\lambda}\lambda}^A \rangle, \langle \hat{T}_{\bar{\psi}\psi} \hat{T}_{\bar{\lambda}\psi} \hat{j}_{\bar{\lambda}\lambda}^A \rangle, \\ & \langle \hat{T}_{\bar{\psi}\psi} \hat{T}_{\bar{\lambda}\lambda} \hat{j}_{\bar{\psi}\psi}^A \rangle, \langle \hat{T}_{\bar{\psi}\lambda} \hat{T}_{\bar{\psi}\psi} \hat{j}_{\bar{\lambda}\lambda}^A \rangle, \langle \hat{T}_{\bar{\psi}\lambda} \hat{T}_{\bar{\lambda}\lambda} \hat{j}_{\bar{\psi}\psi}^A \rangle, \\ & \langle \hat{T}_{\bar{\lambda}\psi} \hat{T}_{\bar{\psi}\psi} \hat{j}_{\bar{\lambda}\lambda}^A \rangle, \langle \hat{T}_{\bar{\lambda}\psi} \hat{T}_{\bar{\lambda}\lambda} \hat{j}_{\bar{\psi}\psi}^A \rangle, \langle \hat{T}_{\bar{\lambda}\lambda} \hat{T}_{\bar{\psi}\psi} \hat{j}_{\bar{\psi}\psi}^A \rangle, \\ & \langle \hat{T}_{\bar{\lambda}\lambda} \hat{T}_{\bar{\psi}\lambda} \hat{j}_{\bar{\psi}\psi}^A \rangle, \langle \hat{T}_{\bar{\lambda}\lambda} \hat{T}_{\bar{\lambda}\psi} \hat{j}_{\bar{\psi}\psi}^A \rangle \rightarrow 0 \quad (m \rightarrow \infty) \end{aligned}$$

Method of conformal correlation functions: gravitational chiral anomaly

Of the 32 correlators, only **9** remain:

$$\begin{aligned}
 \langle T \hat{T}_{\mu\nu}(x) \hat{T}_{\sigma\rho}(y) \hat{j}_{\omega}^A(z) \rangle_c &= \langle \hat{T}_{\bar{\psi}\psi} \hat{T}_{\bar{\psi}\psi} \hat{j}_{\bar{\psi}\psi}^A \rangle \\
 &+ \langle \hat{T}_{\bar{\psi}\psi} \hat{T}_{\bar{\psi}\lambda} \hat{j}_{\bar{\psi}\psi}^A \rangle + \langle \hat{T}_{\bar{\psi}\psi} \hat{T}_{\bar{\lambda}\psi} \hat{j}_{\bar{\psi}\psi}^A \rangle + \langle \hat{T}_{\bar{\psi}\lambda} \hat{T}_{\bar{\psi}\psi} \hat{j}_{\bar{\psi}\psi}^A \rangle \\
 &+ \langle \hat{T}_{\bar{\psi}\lambda} \hat{T}_{\bar{\psi}\lambda} \hat{j}_{\bar{\psi}\psi}^A \rangle + \langle \hat{T}_{\bar{\psi}\lambda} \hat{T}_{\bar{\lambda}\psi} \hat{j}_{\bar{\psi}\psi}^A \rangle + \langle \hat{T}_{\bar{\lambda}\psi} \hat{T}_{\bar{\psi}\psi} \hat{j}_{\bar{\psi}\psi}^A \rangle \\
 &+ \langle \hat{T}_{\bar{\lambda}\psi} \hat{T}_{\bar{\psi}\lambda} \hat{j}_{\bar{\psi}\psi}^A \rangle + \langle \hat{T}_{\bar{\lambda}\psi} \hat{T}_{\bar{\lambda}\psi} \hat{j}_{\bar{\psi}\psi}^A \rangle
 \end{aligned}$$

Of the 9, only **4** are independent:

$$\begin{aligned}
 \langle T \hat{T}_{\bar{\psi}\psi}^{\mu\nu}(x) \hat{T}_{\bar{\psi}\lambda}^{\sigma\rho}(y) \hat{j}_{A\bar{\psi}\psi}^{\omega}(z) \rangle_c &= \langle T \hat{T}_{\bar{\lambda}\psi}^{\mu\nu}(x) \hat{T}_{\bar{\psi}\lambda}^{\sigma\rho}(y) \hat{j}_{A\bar{\psi}\psi}^{\omega}(z) \rangle_c = \\
 = \langle T \hat{T}_{\bar{\psi}\psi}^{\mu\nu}(x) \hat{T}_{\bar{\lambda}\psi}^{\sigma\rho}(y) \hat{j}_{A\bar{\psi}\psi}^{\omega}(z) \rangle_c &= \langle T \hat{T}_{\bar{\psi}\lambda}^{\mu\nu}(x) \hat{T}_{\bar{\lambda}\psi}^{\sigma\rho}(y) \hat{j}_{A\bar{\psi}\psi}^{\omega}(z) \rangle_c, \\
 = -\langle T \hat{T}_{\bar{\psi}\lambda}^{\mu\nu}(y) \hat{T}_{\bar{\psi}\psi}^{\sigma\rho}(x) \hat{j}_{A\bar{\psi}\psi}^{\omega}(z) \rangle_c &= \langle T \hat{T}_{\bar{\lambda}\psi}^{\mu\nu}(x) \hat{T}_{\bar{\lambda}\psi}^{\sigma\rho}(y) \hat{j}_{A\bar{\psi}\psi}^{\omega}(z) \rangle_c = \\
 = -\langle T \hat{T}_{\bar{\lambda}\psi}^{\mu\nu}(y) \hat{T}_{\bar{\psi}\psi}^{\sigma\rho}(x) \hat{j}_{A\bar{\psi}\psi}^{\omega}(z) \rangle_c, &= \langle T \hat{T}_{\bar{\psi}\lambda}^{\mu\nu}(x) \hat{T}_{\bar{\psi}\lambda}^{\sigma\rho}(y) \hat{j}_{A\bar{\psi}\psi}^{\omega}(z) \rangle_c
 \end{aligned}$$

Method of conformal correlation functions: gravitational chiral anomaly

A typical correlator has the form (2 terms - Wick's theorem):

Operator from the axial current

$$\begin{aligned}
 \langle T\hat{T}_{\bar{\psi}\psi}^{\mu\nu}(x)\hat{T}_{\bar{\psi}\psi}^{\sigma\rho}(y)\hat{j}_{A\bar{\psi}\psi}^{\omega}(z)\rangle_c = & \lim_{\substack{x_1, x_2 \rightarrow x \\ y_1, y_2 \rightarrow y \\ z_1, z_2 \rightarrow z}} \left(-\text{tr} \left\{ \mathcal{D}_{(\bar{\psi}\psi)}^{\mu\nu\eta_1\eta_2}(\partial^{x_1}, \partial^{x_2}) G_{\eta_2\eta_5}^{\psi\bar{\psi}}(x_2 - z_1) \mathcal{J}_{A(\bar{\psi}\psi)}^{\omega\eta_5\eta_6} G_{\eta_6\eta_3}^{\psi\bar{\psi}}(z_2 - y_1) \right. \right. \\
 & \times \mathcal{D}_{(\bar{\psi}\psi)}^{\sigma\rho\eta_3\eta_4}(\partial^{y_1}, \partial^{y_2}) G_{\eta_4\eta_1}^{\psi\bar{\psi}}(y_2 - x_1) \left. \right\} - \text{tr} \left\{ \mathcal{D}_{(\bar{\psi}\psi)}^{\mu\nu\eta_1\eta_2}(\partial^{x_1}, \partial^{x_2}) G_{\eta_2\eta_3}^{\psi\bar{\psi}}(x_2 - y_1) \right. \\
 & \left. \left. \times \mathcal{D}_{(\bar{\psi}\psi)}^{\sigma\rho\eta_3\eta_4}(\partial^{y_1}, \partial^{y_2}) G_{\eta_4\eta_5}^{\psi\bar{\psi}}(y_2 - z_1) \mathcal{J}_{A(\bar{\psi}\psi)}^{\omega\eta_5\eta_6} G_{\eta_6\eta_1}^{\psi\bar{\psi}}(z_2 - x_1) \right\} \right)
 \end{aligned}$$

Operators from SET

Green functions

Method of conformal correlation functions: gravitational chiral anomaly

- **How to explain the factor -19?**
- How does it **relate** to **previous** calculations?

$$\langle \nabla_{\mu} \hat{j}_A^{\mu} \rangle_{RS} = \frac{-21}{384\pi^2 \sqrt{-g}} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu\kappa\lambda} R_{\rho\sigma}{}^{\kappa\lambda}$$

[M. J. Duff, 1201.0386] [J.J.M. Carrasco, et al. JHEP, 07:029, 2013]

“ghostless” contribution [J.J.M. Carrasco, et al. JHEP, 07:029, 2013]

$$-19 = -20 + 1$$

$$-19 = -21 + 2$$

Contribution
of spin 1/2

Gauge anomaly

$$5 = 4 + 1$$

$$5 = 3 + 2$$

Method of conformal correlation functions: gravitational chiral anomaly

General Rule will be:

$$\text{Anomaly in RSA} = \text{RS-anomaly} + 2 \cdot (\text{spin } \frac{1}{2} \text{ anomaly})$$

$$\text{Anomaly in RSA} = (\text{RS-anomaly "ghostless"}) + 1 \cdot (\text{spin } \frac{1}{2} \text{ anomaly})$$

Works both for the **gravitational** and the **gauge** chiral anomaly