Backup

The standard $(\beta\beta)_{2\nu}$ is observed for a dozen isotopes with $T^{2\nu}_{1/2} \sim 10^{19-25}$ years. Some most recent averaged/recommended $T^{2\nu}_{1/2}$ are collected in Table and are compared with theoretical predictions.

		$T^{2\nu}_{1/2}$ (years)	
Element	Isotope	Measured	Calculated
Calcium	$^{48}_{20}$Ca	$5.3^{+1.2}_{-0.8} \times 10^{19}$	$6 \times 10^{18} - 5 \times 10^{20}$
Germanium	$^{76}_{32}$Ge	$(1.88 \pm 0.08) \times 10^{21}$	$7 \times 10^{19} - 6 \times 10^{22}$
Selenium	$^{82}_{34}$Se	$8.7^{+0.2}_{-0.1} \times 10^{19}$	$3 \times 10^{18} - 6 \times 10^{21}$
Zirconium	$^{96}_{40}$Zr	$(2.3 \pm 0.2) \times 10^{19}$	$3 \times 10^{17} - 6 \times 10^{20}$
Molybdenum	$^{100}_{42}$Mo	$7.06^{+0.15}_{-0.12} \times 10^{18}$	$1 \times 10^{17} - 2 \times 10^{22}$
Molybdenum–Ruthenium	$^{100}_{42}$Mo$-^{100}_{44}$Ru(0^+_1)	$6.7^{+0.5}_{-0.4} \times 10^{20}$	$5 \times 10^{19} - 2 \times 10^{21}$
Cadmium	$^{116}_{48}$Cd	$(2.69 \pm 0.09) \times 10^{19}$	$3 \times 10^{18} - 2 \times 10^{21}$
Tellurium	$^{128}_{52}$Te	$(2.25 \pm 0.09) \times 10^{24}$	$9 \times 10^{22} - 3 \times 10^{25}$
Tellurium	$^{130}_{52}$Te	$(7.91 \pm 0.21) \times 10^{20}$	$2 \times 10^{19} - 7 \times 10^{20}$
Xenon	$^{136}_{54}$Xe	$(2.18 \pm 0.05) \times 10^{21}$	–
Neodymium	$^{150}_{60}$Nd	$(9.34 \pm 0.65) \times 10^{18}$	$6 \times 10^{16} - 4 \times 10^{20}$
Neodymium–Samarium	$^{150}_{60}$Ne$-^{150}_{62}$Sm(0^+_1)	$1.2^{+0.3}_{-0.2} \times 10^{20}$	–
Uranium	$^{238}_{92}$U	$(2.0 \pm 0.6) \times 10^{21}$	$2 \times 10^{19} - 2 \times 10^{23}$

[From A. S. Barabash, "Precise half-life values for two-neutrino double-β decay: 2020 review," Universe **6** (2020) 159, arXiv:2009.14451 [nucl-ex] (experiment); E. Fiorini, "Experimental prospects of neutrinoless double beta decay," Phys. Scripta **T121** (2005) 86–93 (theory; of course these calculations are outdated, but I did not find a fresh review).]

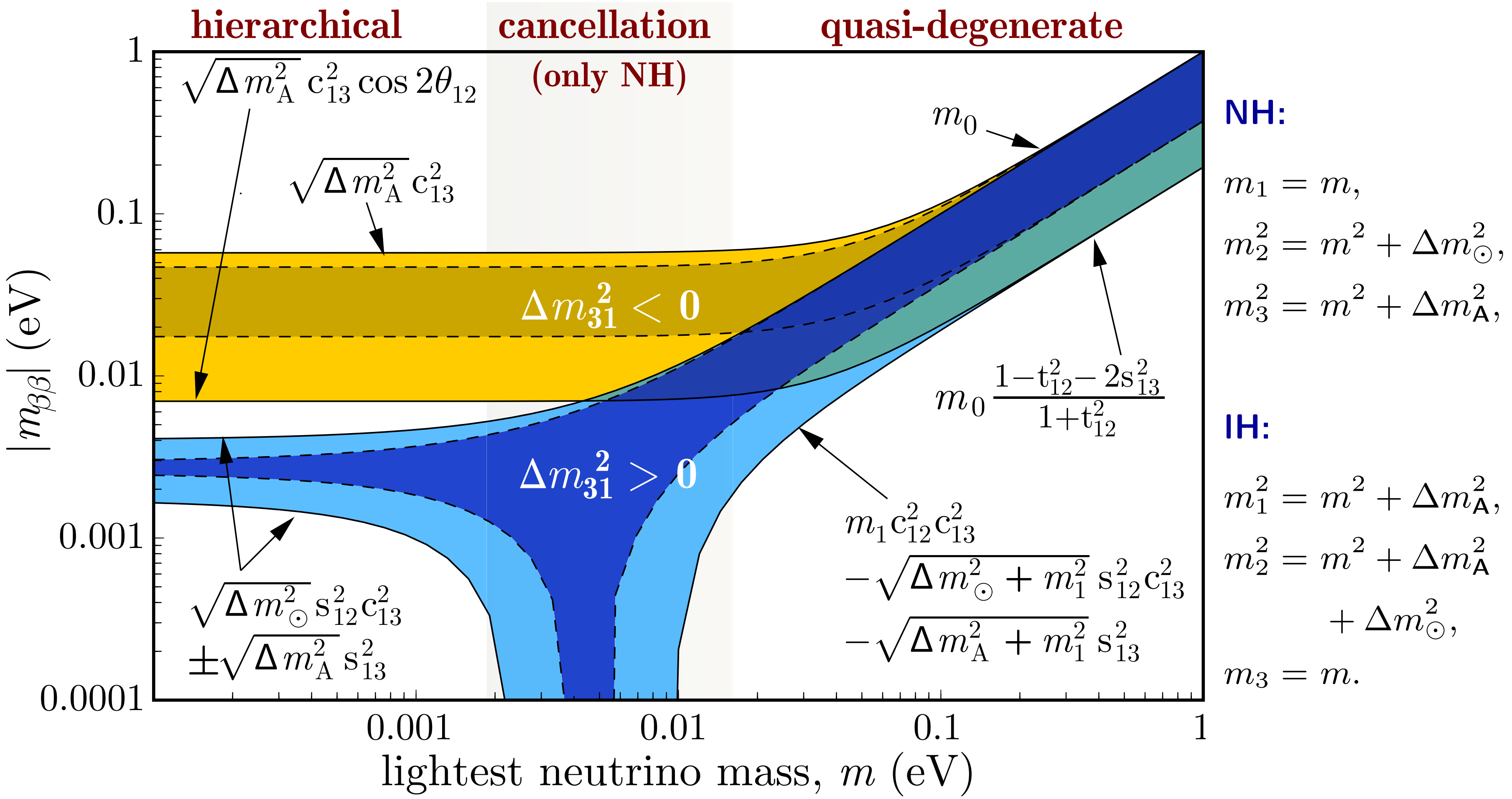
Best current results on $0\nu\beta\beta$ decay. The $T^{0\nu}_{1/2}$ and $\langle m_{\beta\beta}\rangle(\equiv \langle|m_{\beta\beta}|\rangle)$ limits are at 90% C.L.

Element	Isotope	$Q_{2\beta}$ (keV)	$T^{0\nu}_{1/2}$ (years)	$\langle m_{\beta\beta}\rangle$ (eV)	Experiment
Calcium	^{48}Ca	4267.98	$> 5.8 \times 10^{22}$	$< 3.5 - 22$	ELEGANT-IV
Germanium	^{76}Ge	2039.00	$> 8.0 \times 10^{25}$	$< 0.12 - 0.26$	GERDA
			$> 1.9 \times 10^{25}$	$< 0.24 - 0.52$	Majorana Demonstrator
Selenium	^{82}Se	2997.9	$> 3.6 \times 10^{23}$	$< 0.89 - 2.4$	NEMO-3
Zirconium	^{96}Zr	3355.85	$> 9.2 \times 10^{21}$	$< 7.2 - 19.5$	NEMO-3
Molybdenum	^{100}Mo	3034.40	$> 1.1 \times 10^{24}$	$< 0.33 - 0.62$	NEMO-3
Cadmium	^{116}Cd	2813.50	$> 2.2 \times 10^{23}$	$< 1.0 - 1.7$	AURORA
Tellurium	^{128}Te	866.6	$> 1.1 \times 10^{24}$	–	Geochemical
Tellurium	^{130}Te	2527.52	$> 1.5 \times 10^{25}$	$< 0.11 - 0.52$	CUORE
Xenon	^{136}Xe	2457.83	$> \mathbf{1.07 \times 10^{26}}$	$< \mathbf{0.061 - 0.165}$	KamLAND-Zen
			$> 1.8 \times 10^{25}$	$< 0.15 - 0.40$	EXO-200
Neodymium	^{150}Nd	3371.38	$> 2.0 \times 10^{22}$	$< 1.6 - 5.3$	NEMO-3

The $\langle m_{\beta\beta}\rangle$ limits are listed as reported in the original publications.[a]

[M. J. Dolinski, A. W. P. Poon, & W. Rodejohann, "Neutrinoless double-beta decay: Status and prospects," Ann. Rev. Nucl. Part. Sci. **69** (2019) 219–251, arXiv:1902.04097 [nucl-ex].]

[a]For a bit another approach, see A. S. Barabash, "Brief review of double beta decay experiments", arXiv:1702.06340 [nucl-ex]; the Q values shown in the Table are borrowed from that paper.

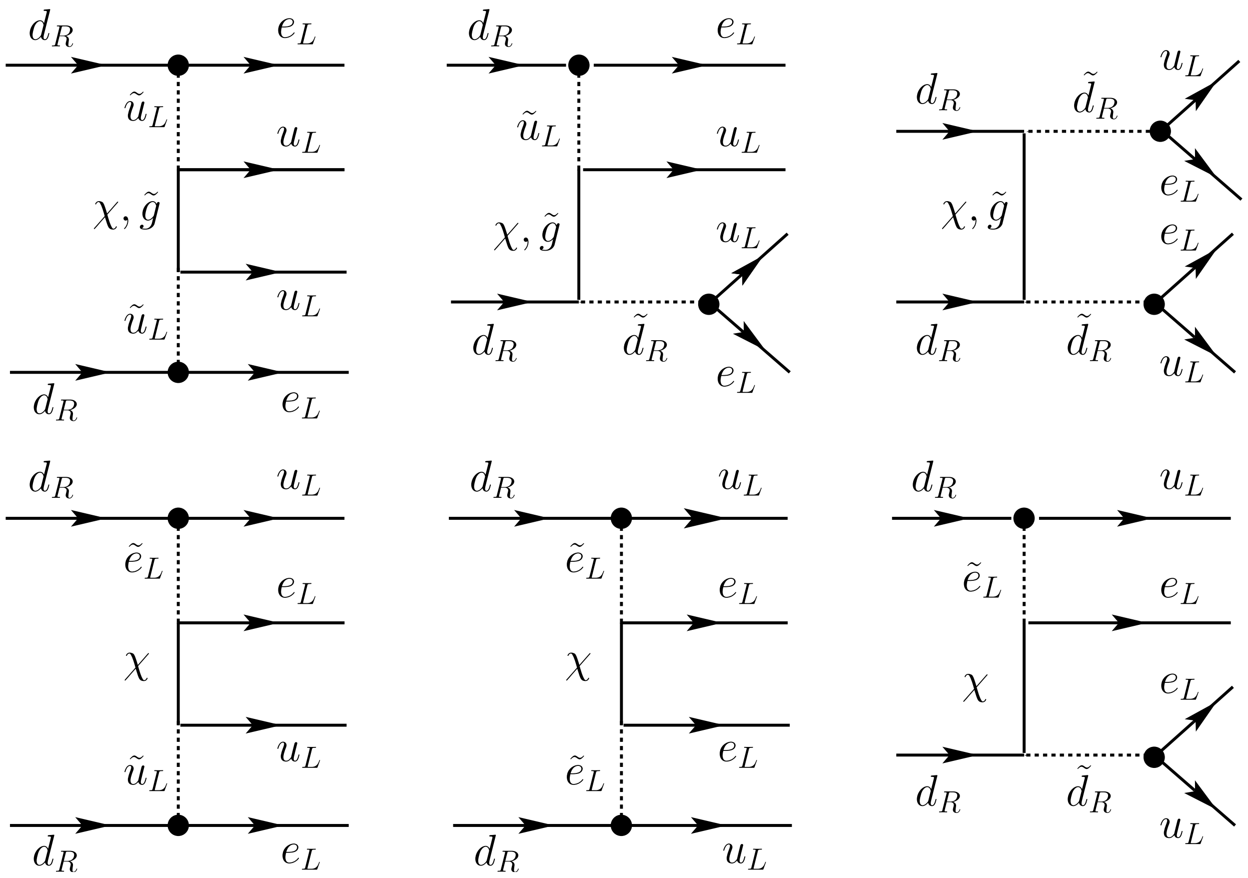


The main properties of $|m_{\beta\beta}|$ vs. smallest neutrino mass (m). The value of $\sin^2 2\theta_{13} = 0.02$ has been chosen, m_0 is the common mass scale (measurable in KATRIN or by cosmology via $\sum_i m_i/3$) for quasi-degenerate masses $m_1 \simeq m_2 \simeq m_3 \equiv m_0 \gg \sqrt{\Delta m^2_A}$ (corrections are small as $m \gtrsim 0.03$ eV).

[Taken from M. Lindner, A. Merle, and W. Rodejohann,“Improved limit on θ_{13} and implications for neutrino masses in neutrinoless double beta decay and cosmology,” Phys. Rev. D **73** (2006) 053005, hep-ph/0512143.]

Schechter-Valle (black-box) theorem.

Current particle models (GUTs, R-parity violating SUSY, etc.) provide mechanisms, other than neutrino mass, which can contribute to or even dominate the $0\nu\beta\beta$ process (see example below).



$\triangle$ R-parity violating contribution to $0\nu\beta\beta$ decay mediated by sfermions and neutralinos (gluinos).

[Figure is borrowed from J. D. Vergados, H. Ejiri, and F. Simkovic, "Theory of neutrinoless double-beta decay," Rep. Prog. Phys. **75** (2012) 106301, arXiv:1205.0649 [hep-ph], where many other examples can be found.]

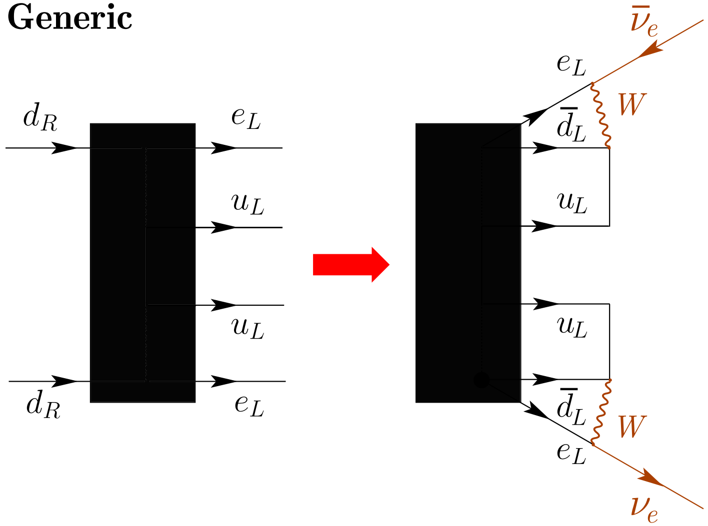
Schechter and Valle proved[a] that

for any realistic gauge theory including the usual (SM) W-gauge-field interaction with left-handed e and ν_e and with u and d quarks, if $0\nu\beta\beta$-decay takes place, regardless of the mechanism causing it, the neutrino is Majorana particle with nonzero mass.

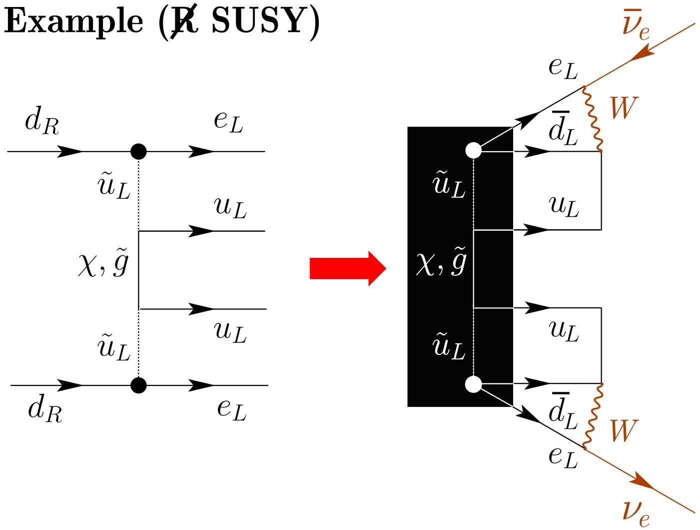
The reason is that one can consider the $0\nu\beta\beta$ elementary interaction process $dd \to uuee$ as generated by the black box, which can include any mechanism. Then the legs of the black box can be arranged to form a diagram which generates $\overline{\nu}_e \to \nu_e$ transitions. This diagram contributes to the Majorana mass of the electron neutrino through radiative corrections at some order of perturbation theory, even if there is no tree-level Majorana neutrino mass term.

It is however clear that the black-box amplitudes are strongly suppressed (at least by a factor $\propto G_F^2$) with respect to the standard tree-level $0\nu\beta\beta$-decay amplitude. Model calculations show that the standard amplitude corresponding to a value of $|m_{\beta\beta}| = O(0.1)$ eV generates radiatively a Majorana mass $O(10^{-24})$ eV.

Generic



Example ($\not R$ SUSY)



[a] J. Schechter and J. W. F. Valle, "Neutrinoless double-β decay in SU(2)$\times$U(1) theories," Phys. Rev. D **25** (1982) 2951–2954. A generalization to 3ν (mixed) case was made by M. Hirsch, H. V. Klapdor-Kleingrothaus, and S. G. Kovalenko, "On the SUSY accompanied neutrino exchange mechanism of neutrinoless double beta decay," Phys. Lett. B **372** (1996) 181–186, Phys. Lett. B **381** (1996) 488 (erratum), hep-ph/9512237.

7.10 Double see-saw & inverse see-saw.

The see-saw can be implemented by introducing additional neutrino singlets beyond the three RH neutrinos involved into the see-saw type I. One have to distinguish between

- RH neutrinos $\boldsymbol{\nu}_R$, which carry $B-L$ and perhaps (not necessary) form $SU(2)_R$ doublets with RH charged leptons, and
- Neutrino singlets $\boldsymbol{\nu}_S$, which have no Yukawa couplings to the LH neutrinos but may couple to $\boldsymbol{\nu}_R$.

If the singlets have nonzero Majorana masses $\mathbf{M}_{SS}$ while the RH neutrinos have a zero Majorana mass, $\mathbf{M}_{RR}=0$, the see-saw mechanism may proceed via mass couplings of the singlets to RH neutrinos, $\mathbf{M}_{RS}$. In the basis $(\boldsymbol{\nu}_L,\boldsymbol{\nu}_R,\boldsymbol{\nu}_S)$, the 9×9 mass matrix is

$$\begin{pmatrix} \mathbf{0} & \mathbf{m}_{LR} & \mathbf{0} \\ \mathbf{m}_{LR} & \mathbf{0} & \mathbf{M}_{RS} \\ \mathbf{0} & \mathbf{M}_{RS}^T & \mathbf{M}_{SS} \end{pmatrix}.$$

Assuming that the eigenvalues of $\mathbf{M}_{SS}$ are much smaller than the eigenvalues of $\mathbf{M}_{RS}$, the light physical LH Majorana neutrino masses are then doubly suppressed,

$$\mathbf{M}_1 \simeq \mathbf{m}_{LR}\mathbf{M}_{RS}^{-1}\mathbf{M}_{SS}\left(\mathbf{M}_{RS}^T\right)^{-1}\mathbf{m}_{LR}^T, \quad \mathbf{M}_2^2 \simeq \mathbf{M}_{RS}^2+\mathbf{m}_{LR}^2.$$

This scenario is usually used in string inspired models [see, e.g., R.N.Mohapatra & J.W.Valle, Phys. Rev. D **34** (1986) 1642; M.C.Gonzalez-Garcia & J.W.F.Valle, Phys. Lett. B **216** (1989) 360].

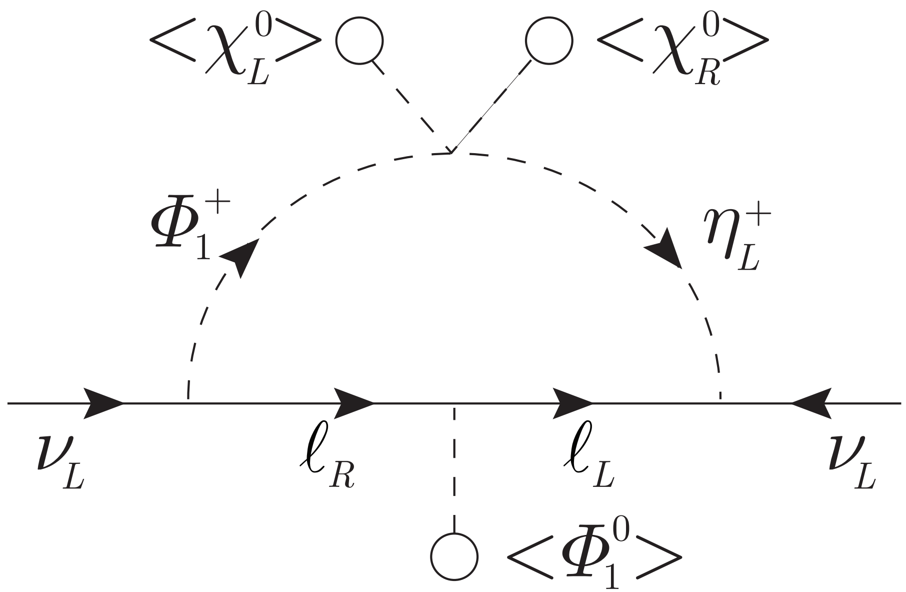
7.11 Radiative see-saw.

An alternative mechanism relies on the radiative generation of neutrino masses [H.Georgi & S.L.Glashow, Phys. Rev. D **7** (1973) 2487; P.Cheng & L.-F.Li, Phys. Rev. D **17** (1978) 2375; Phys. Rev. D **22** (1980) 2860; A.Zee, Phys. Lett. B **93** (1980) 389;. . ..] In this scheme, the neutrinos are massless at the tree level, but pick up small masses due to loop corrections.

In a typical model [K.S. Babu & V.S. Mathur, Phys. Rev. D **11** (1988) 3550] the see-saw formula is modified as

$$m_\nu \sim \left(\frac{\alpha}{\pi}\right)\frac{m_l^2}{M},$$

where the prefactor $\alpha/\pi \approx 2\times 10^{-3}$ arises due to the loop structure of the neutrino mass diagram. Light neutrinos are now possible even for relatively "light" mass scale M of "new physics."



The scalar sector consists of the multiplets

$$\chi_{L,R} = \left(\chi^+,\chi^0\right)_{L,R}, \quad \Phi = \begin{pmatrix}\Phi_1^0 & \Phi_2^+ \\ \Phi_1^- & \Phi_2^0\end{pmatrix}, \quad \eta_{L,R}^+.$$

The diagram in the figures is responsible for generation of Majorana masses for ν_L. The analogous diagram is obtained by the replacement $L \to R$ and $\Phi_1^+ \to \Phi_2^+$.

7.12 TeV-scale gauged $B-L$ symmetry with Inverse see-saw.

Consider briefly one more inverse see-saw model [S.Khalil, Phys. Rev. D **82** (2010) 077702].

The model is based on the following:

(i) The SM singlet Higgs boson, which breaks the $B-L$ gauge symmetry, has $B-L$ unit charge.

(ii) The SM singlet fermion sector includes two singlet fermions $S_{\pm}$ with $B-L$ charges ± 2 with opposite matter parity.

The Lagrangian of neutrino masses, in the flavor basis, is given by

$$\overline{\boldsymbol{\nu}}_L \mathbf{m}_D \boldsymbol{\nu}_R + \boldsymbol{\nu}_R^c \mathbf{M}_N S_- + \mu_s \overline{\boldsymbol{S}}_- \boldsymbol{S}_-.$$

In the limit $\mu_s \to 0$, which corresponds to the unbroken $(-1)^{L+S}$ symmetry, the light neutrinos remain massless. Therefore, a small nonvanishing μ_s can be considered as a slight breaking of a this global symmetry and the smallness of μ_s is natural. Small μ_s can also be generated radiatively.

In the basis $(\boldsymbol{\nu}_L, \boldsymbol{\nu}_R^c, \boldsymbol{S}_-)$, the 9×9 mass matrix is

$$\begin{pmatrix} \mathbf{0} & \mathbf{m}_D & \mathbf{0} \\ \mathbf{m}_D^T & \mathbf{0} & \mathbf{M}_N \\ \mathbf{0} & \mathbf{M}_N^T & \mu_s \end{pmatrix}.$$

So, up to the notation, it reproduces all the properties of the double see-saw.