$\mathcal{N}=2$ higher spin theories in harmonic superspace

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Abstract

We present off-shell unconstrained formulation of $\mathcal{N}=2$ Fronsdal theory and their cubic $(\frac{1}{2},\frac{1}{2},\mathrm{s})$ couplings to hypermultiplet using $\mathcal{N}=2$ harmonic superspace approach.

Harmonic superspace

4D $\mathcal{N}=2$ superspace defined as coset:

$$\mathbb{R}^{4|8}=\frac{\{M_{ab},P_a,Q^i_\alpha,\bar{Q}^i_{\dot\alpha},su(2)\}}{\{M_{ab}\},su(2)}=(x^a,\theta^i_\alpha,\bar\theta^i_{\dot\alpha})\,.$$

4D $\mathcal{N}=2$ harmonic superspace can also be defined as coset:

$$\mathbb{HR}^{4+2|8}=\frac{\{M_{ab},P_a,Q^i_\alpha,\bar{Q}^i_{\dot\alpha},su(2)\}}{\{M_{ab},u(1)\}}=\mathbb{R}^{4|8}\times S^2=(x^a,\theta^i_\alpha,\bar\theta^i_{\dot\alpha},u^{\pm i})\,.$$

- Harmonic superfields $Q^{(n)}(x^a,\theta^i_\alpha,\bar\theta^i_{\dot\alpha},u^{\pm i})$ have infinitely many components.
- Harmonic superspace have new invariant subspace containing only half of the original Grassmann variables. Analytic superspace (analog of chiral superspace in $\mathcal{N}=1, d=4$):

$$\mathbb{HA}^{4+2|4}=\frac{\{M_{ab},P_a,Q^i_\alpha,\bar{Q}^i_{\dot\alpha},su(2)\}}{\{M_{ab},Q^+_\alpha,\bar{Q}^+_{\dot\alpha},u(1)\}}=(x^a,\theta^+_\alpha,\theta^+_{\dot\alpha},u^{\pm i})=\zeta_A\,.$$

- Harmonic superspace allow off-shell hypermultiplet:

$$S_{hyper}=-\frac{1}{2}\int d^4xd^4\theta^+du\;q^{+a}\mathcal{D}^{++}q^+_a=-\int d^4xd^4\theta^+du\;\tilde{q}^+\mathcal{D}^{++}q^+\,. \quad (1)$$

- Here $q^+_a(x,\theta^+,u)=(q^+,-\tilde{q}^+)$, $q^{+a}=\epsilon^{ab}q^+_b$. q^+ is unconstrained analytic superfield with component expansion:

$$q^+(x^a,\theta^+,u)=f^i(x)u^+_i+\theta^{+\alpha}\psi_\alpha+\bar\theta_{\dot\alpha}\bar\kappa^{\dot\alpha}+\dots\,.$$

- Harmonic derivative:

$$\mathcal{D}^{++}=\partial^{++}-2i\theta^{+\rho}\bar\theta^{+\dot\rho}\partial_{\rho\dot\rho}+\theta^{+\hat\mu}\partial^+_{\hat\mu}+i(\hat{\theta^+})^2\partial_5\,,\quad \partial^{++}=u^{+i}\frac{\partial}{\partial u^{-i}}\,,\quad \hat\mu=(\mu\,,\dot\mu)\,.$$

  Here x^5 is auxiliary coordinate and is used for description of supermultiplets in the presence of central charge, $\partial_5 q^+:=imq^+$.
- Hypermultiplet equations of motion after exclusion of auxiliary fields gives:

$$\mathcal{D}^{++}q^+=0\quad\Rightarrow\quad(\Box+m^2)f^i=0\,,\quad i\partial_{\alpha\dot\alpha}\bar\kappa^{\dot\alpha}+m\psi_\alpha=0\,,\quad i\partial_{\alpha\dot\alpha}\psi^\alpha-m\bar\kappa_{\dot\alpha}=0\,.$$

- Unconstrained analytic prepotential of **$\mathcal{N}=2$ Maxwell multiplet** appear as connection in harmonic derivative:

$$\mathcal{D}^{++}\quad\Rightarrow\quad\mathcal{D}^{++}+iV^{++}\,. \quad (2)$$

- Unconstrained analytic prepotentials of **$\mathcal{N}=2$ supergravity** appear as vielbeins in harmonic derivative:

$$\mathcal{D}^{++}\quad\Rightarrow\quad\mathfrak{D}^{++}=\mathcal{D}^{++}+h^{++\mu\dot\mu}\partial_{\mu\dot\mu}+h^{++\hat\mu+}\partial^-_{\hat\mu}+h^{++5}\partial_5\,. \quad (3)$$

$\mathcal{N}=2$ spin 1 theory

- Using gauge transformations in Abelian $\mathcal{N}=2$ gauge theory, one can impose Wess-Zumino gauge:

$$\delta V^{++}=\mathcal{D}^{++}\Lambda\;\Rightarrow\;V^{++}=(\theta^+)^2\phi+(\bar\theta^+)^2\bar\phi+2i\theta^{+\alpha}\bar\theta^{+\dot\alpha}A_{\alpha\dot\alpha}+(\bar\theta^+)^2\theta^{+\alpha}\psi^i_\alpha u^-_i$$
$$+(\theta^+)^2\bar\theta^+_{\dot\alpha}\bar\psi^{\dot\alpha i}u^-_i+(\theta^+)^2(\bar\theta^+)^2D^{(ik)}u^-_iu^-_k\,.$$

- 4D fields ϕ, $\bar\phi$, $A_{\alpha\dot\alpha}$, ψ^i_α, $\bar\psi^i_{\dot\alpha}$, $D^{(ik)}$ constitute an Abelian gauge $\mathcal{N}=2$ off-shell multiplet ($8+8$ off-shell degrees of freedom). On-shell we have the physical field multiplet $(\mathbf{1},\mathbf{1/2},\mathbf{1/2},\mathbf{0})$.

- Gauge invariant action:

$$S\sim\int d^4xd^8\theta du\;V^{++}V^{--}\,,$$

  where V^{--} is the solution of zero curvature equation:

$$\mathcal{D}^{++}V^{--}-\mathcal{D}^{--}V^{++}=0\,,\qquad\delta V^{--}=\mathcal{D}^{--}\Lambda\,,$$
$$\mathcal{D}^{--}=\partial^{--}-2i\theta^{-\rho}\bar\theta^{-\dot\rho}\partial_{\rho\dot\rho}+\theta^{-\hat\mu}\partial^-_{\hat\mu}+i(\hat{\theta^-})^2\partial_5\,,\quad\partial^{--}=u^{-i}\frac{\partial}{\partial u^{+i}}\,.$$

$\mathcal{N}=2$ spin 2 theory

- Analogs of V^{++} are the set of analytic gauge prepotentials $(h^{++\alpha\dot\alpha}\,,\;h^{++5}\,,\;h^{++\hat\mu+})$ in harmonic derivative $\mathfrak{D}^{++}$ (3) with gauge transformations:

$$\delta_\lambda h^{++\alpha\dot\alpha}=D^{++}\lambda^{\alpha\dot\alpha}+2i\big(\lambda^{+\alpha}\bar\theta^{+\dot\alpha}+\theta^{+\alpha}\bar\lambda^{+\dot\alpha}\big)\,,$$
$$\delta_\lambda h^{++5}=D^{++}\lambda^5-2i\big(\lambda^{+\alpha}\theta^+_\alpha-\bar\theta^+_{\dot\alpha}\bar\lambda^{+\dot\alpha}\big),$$
$$\delta_\lambda h^{++\hat\mu+}=D^{++}\lambda^{+\hat\mu}\,.$$

- Wess-Zumino gauge:

$$h^{++m}=-2i\theta^+\sigma^a\bar\theta^+\Phi^m_a+\big[(\bar\theta^+)^2\theta^+\psi^{m\,i}u^-_i+(\theta^+)^2\bar\theta\bar\psi^{mi}u^-_i\big]+\dots$$
$$h^{++5}=-2i\theta^+\sigma^a\bar\theta^+C_a+\dots\,,\qquad h^{++\hat\mu+}=\dots\,.$$

- The physical fields are $\Phi^m_a,\psi^{m\,i}_\mu,C_a$ (Supermultiplet of $\mathcal{N}=2$ Einstein supergravity $(\mathbf{2},\mathbf{3/2},\mathbf{3/2},\mathbf{1})$ on shell).
- The invariant action:

$$S\sim\int d^4xd^8\theta du\;\Big(G^{++\alpha\dot\alpha}G^{--}_{\alpha\dot\alpha}+G^{++5}G^{--5}\Big),$$

  Here we used composite superfields:

$$G^{++\mu\dot\mu}:=h^{++\mu\dot\mu}+2i\big(h^{++\mu+}\bar\theta^{-\dot\mu}+\theta^{-\mu}h^{++\dot\mu+}\big)\,,$$
$$G^{++5}:=h^{++5}-2i\big(h^{++\mu+}\theta^-_\mu-\bar\theta^-_{\dot\mu}h^{++\dot\mu+}\big)\,,$$
$$\mathcal{D}^{++}G^{--\mu\dot\mu}=\mathcal{D}^{--}G^{++\mu\dot\mu}\,,\qquad\mathcal{D}^{++}G^{--5}=\mathcal{D}^{--}G^{++5}\,.$$

$\mathcal{N}=2$ spin s theory

- The general case with the maximal spin s is spanned by the following analytic gauge prepotentials:

$$h^{++\alpha(s-1)\dot\alpha(s-1)}\,,\;h^{++\alpha(s-2)\dot\alpha(s-2)}\,,\;h^{++\alpha(s-1)\dot\alpha(s-2)+}\,,\;h^{++\dot\alpha(s-1)\alpha(s-2)+}\,,$$

  where $\alpha(s):=(\alpha_1\dots\alpha_s),\dot\alpha(s):=(\dot\alpha_1\dots\dot\alpha_s)$.
- The relevant gauge transformations can also be defined and shown to leave, in the WZ-like gauge, the physical field multiplet $(\mathbf{s},\mathbf{s-1/2},\mathbf{s-1/2},\mathbf{s-1})$.
- The invariant action has the universal form for any s

$$S_{(s)}=(-1)^{s+1}\int d^4xd^8\theta du\,\Big\{G^{++\alpha(s-1)\dot\alpha(s-1)}G^{--}_{\alpha(s-1)\dot\alpha(s-1)}$$
$$+G^{++\alpha(s-2)\dot\alpha(s-2)}G^{--}_{\alpha(s-2)\dot\alpha(s-2)}\Big\},$$

  where G-superfields are defined analogously to spin **2** case.

In Wess-Zumino gauge, one can check that in components presented actions give corresponding Fronsdal and Fang-Fronsdal actions. A detailed discussion of component reduction and solutions of zero curvature equations is given in [1].

Hypermultiplet couplings

Here we present cubic $(\mathbf{1/2},\mathbf{1/2},\mathbf{s})$ vertices. Their form is fully fixed by supergauge transformations and $\mathcal{N}=2$ rigid supersymmetry.

- Spin **1** hypermultiplet coupling:

$$S_{spin\;1}=-\int d^4xd^4\theta^+du\;\tilde{q}^+\left(\mathcal{D}^{++}+iV^{++}\right)q^+\,.$$

- Spin **2** hypermultiplet coupling:

$$\hat{\mathcal{H}}^{++}_{(2)}:=h^{++\alpha\dot\alpha}\partial_{\alpha\dot\alpha}+h^{++\hat\mu+}\partial^-_{\hat\mu}+h^{++5}\partial_5\,,$$
$$S_{spin\;2}=-\frac{1}{2}\int d^4xd^4\theta^+du\;q^{+a}\left(\mathcal{D}^{++}+\hat{\mathcal{H}}^{++}_{(2)}\right)q^+_a\,. \quad (4)$$

- General spin s hypermultiplet couplings are different for odd and even spins:

$$\hat{\mathcal{H}}^{++}_{(s)}:=\Big(h^{++\alpha(s-1)\dot\alpha(s-1)}\partial_{\alpha\dot\alpha}+h^{++\alpha(s-1)\dot\alpha(s-2)+}\partial^-_\alpha$$
$$+h^{++\alpha(s-2)\dot\alpha(s-1)+}\partial^-_{\dot\alpha}+h^{++\alpha(s-2)\dot\alpha(s-2)}\partial_5\Big)\partial^{(s-2)}_{\alpha(s-2)\dot\alpha(s-2)}\,,$$

$$S_{even\;spin\;s}=-\frac{1}{2}\int d^4xd^4\theta^+du\;q^{+a}\left(\mathcal{D}^{++}+\hat{\mathcal{H}}^{++}_{(s)}\right)q^+_a\,,$$

$$S_{odd\;spin\;s}=-\frac{1}{2}\int d^4xd^4\theta^+du\;q^{+a}\left(\mathcal{D}^{++}+\hat{\mathcal{H}}^{++}_{(s)}J\right)q^+_a\,,\qquad Jq^{+a}=i(\tau_3)^a_bq^{+b}\,.$$

Form of this couplings fully resembled structure of the hypermultiplet coupling to $\mathcal{N}=2$ supergravity (4) and can be constructed by gauging of global "higher spin" symmetries of hypermultiplet action (1). For the future details see [2].

References

[1] I. Buchbinder, E. Ivanov and N. Zaigraev, *Unconstrained off-shell superfield formulation of 4D, $\mathcal{N}=2$ supersymmetric higher spins*, JHEP **12** (2021), 016 [arXiv:2109.07639 [hep-th]].

[2] I. Buchbinder, E. Ivanov and N. Zaigraev, *Off-shell cubic hypermultiplet couplings to $\mathcal{N}=2$ higher spin gauge superfields*, JHEP **05** (2022), 104 [arXiv:2202.08196 [hep-th]].