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MODEL-INDEPENDENT CONSTRAINTS ON PHYSICAL PARAMETERS OF EXTRA NEUTRAL HEAVY BOSON AT THE ILC

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1 Introduction

Today, the Standard Model (SM) is considered as a low-energy approximation of the future fundamental theory describing all interactions. The SM is consistent with almost all experimental data, but also has obvious shortcomings, that are the reason for further more detailed verification of the model and the search for "new" physics

In the presence of non-standard physics in nature, future accelerator experiments should have deviations from the behavior of the SM. If the deviation is significant, it can be interpreted using the parameters of massive gauge Z'-boson.

The gauge group of a typical model that predict one additional boson Z' has the form:

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U'(1), \tag{1}$$

where the SM is complemented by a $U'(1)$ gauge group.

The symmetry of a $U'(1)$ gauge group is broken at the energy of the order of TeV, as a result of which the heavy Z' boson production is possible.

For finding deviations and obtaining constraints on the parameters of Z', lepton colliders have a significant advantage: various deviations can be detected due to a small background. As experiments LEP and SLC have shown, the final fermion states μ, τ, c, b can be detected. Fermion pair production have a unique property: all couplings of the Z' to fermions can be constrained separately.

Therefore, the process is used to extract constraints on the parameters of Z':

$$e^+e^- \to \gamma, \mathrm{Z}^0, \mathrm{Z}' \to f\bar{f}, \tag{2}$$

where $f \neq e$.

Due to the small background, high energy and the possibility of e^+ and e^- beam polarization the future e^+e^- colliders ILC, CLIC and FCC-ee will allow us to study the scales and scenarios of "new" physics that are not available to the Large Hadron Collider (LHC).

The current limits on the mass of Z' (~ 5 TeV) are noticeably larger compared to the planned energy of the next-to-launch e^+e^- collider – ILC (1 TeV). Therefore, it is possible to study only indirect effects manifested in the form of deviation of the observable from behavior in the SM. The experimental information for this case can be represented in the form of constraints on the Z' parameters. The obtained constraints are useful for correcting of the Z' models and for constructing a future fundamental theory.

2 Differential cross section

For the model-independent analysis, it is necessary to obtain a differential cross section containing generalized, effective Z' parameters linearly included in the cross section. Linearity is necessary for opportunity of obtain constraints provided that the deviation from the SM is no more than one standard deviation.

For this purpose, the differential cross section of scattering for the process (2) in the Born approximation can be represented as:

$$\frac{d\sigma^{\mathrm{SM+Z'}}}{dz}(P_{e^-}, P_{e^+}) = N_C(1 - P_{e^-}P_{e^+})\frac{\alpha^2\beta\pi}{8s}\times$$
$$\times\left[(1-z\beta)^2 q_1^{\mathrm{SM+Z'}} + (1+z\beta)^2 q_2^{\mathrm{SM+Z'}} + \eta_f^2 q_3^{\mathrm{SM+Z'}}\right] = \frac{d\sigma^{\mathrm{SM}}}{dz} + \frac{\Delta d\sigma^{\mathrm{Z'}}}{dz} . \tag{3}$$

In the formula (3): $z \equiv \cos\theta$ (θ is the angle between e^- and f); N_C is the color coefficient ($N_C = 1(3)$ for $f = l(q)$); α is the fine-structure constant; P_{e^+} and P_{e^-} are the degrees of longitudinal polarization e^+ and e^- beam; $\beta = \sqrt{1 - 4m_f^2/s}$ ($\eta = \sqrt{1-\beta^2}$); m_f is the mass of the final fermion; $\sqrt{s}$ is the collision energy.

$q_{1,2,3}^{\mathrm{SM+Z'}}$ parameters are determined by $q_{\lambda_e,\lambda_f}^{\mathrm{SM+Z'}}$ combinations (λ_e and λ_f is the helicity of the initial and final state) and function $P_{\mathrm{eff}} = (P_{e^-} - P_{e^+})/(1 - P_{e^-}P_{e^+})$:

$$\begin{aligned} q_1^{\mathrm{SM+Z'}} &= p_{\mathrm{eff}}^-|q_{LR}^{\mathrm{SM+Z'}}|^2 + p_{\mathrm{eff}}^+|q_{RL}^{\mathrm{SM+Z'}}|^2 , \\ q_2^{\mathrm{SM+Z'}} &= p_{\mathrm{eff}}^-|q_{LL}^{\mathrm{SM+Z'}}|^2 + p_{\mathrm{eff}}^+|q_{RR}^{\mathrm{SM+Z'}}|^2 , \\ q_3^{\mathrm{SM+Z'}} &= 2p_{\mathrm{eff}}^-\,\Re[q_{LL}^{\mathrm{SM+Z'}} q_{LR}^{*\mathrm{SM+Z'}}] + 2p_{\mathrm{eff}}^+\,\Re[q_{RL}^{\mathrm{SM+Z'}} q_{RR}^{*\mathrm{SM+Z'}}] , \end{aligned} \tag{4}$$

where $p_{\mathrm{eff}}^{\pm} = 1 \pm P_{\mathrm{eff}}$.

In turn, $q_{\lambda_e,\lambda_f}^{\mathrm{model}}$ parameters containing the physical characteristics of the Z'-boson (couplings, mass and total width), are determined by the:

$$\begin{aligned} q_{LL}^{\mathrm{SM+Z'}} &= \sum_i \frac{s g_{i,e}^L g_{i,f}^L}{s - m_i^2 + \mathrm{i}m_i\Gamma_i} , \quad q_{RR}^{\mathrm{SM+Z'}} = \sum_i \frac{s g_{i,e}^R g_{i,f}^R}{s - m_i^2 + \mathrm{i}m_i\Gamma_i}, \\ q_{LR}^{\mathrm{SM+Z'}} &= \sum_i \frac{s g_{i,e}^L g_{i,f}^R}{s - m_i^2 + \mathrm{i}m_i\Gamma_i} , \quad q_{RL}^{\mathrm{SM+Z'}} = \sum_i \frac{s g_{i,e}^R g_{i,f}^L}{s - m_i^2 + \mathrm{i}m_i\Gamma_i} , \end{aligned} \tag{5}$$

where $g_{i,f}^{L,R} \equiv g_{i,f}^{\mp}$ are the fermion couplings with $i = \gamma, \mathrm{Z}^0, \mathrm{Z}'$ bosons with corresponding masses m_i and decay widths Γ_i.

The fermion couplings with Z^0 and γ are determined by the values of electric charges Q_f and third components of the isospin t_f:

$$g_{\mathrm{Z}^0,f}^{\rho} = \left(\delta_{\rho,-}t_f/2 - Q_f s_{\mathrm{w}}^2\right)/(s_{\mathrm{w}}c_{\mathrm{w}}) , g_{\gamma,f}^{\rho} = -Q_f , \quad \rho = \mp , \tag{6}$$

where $s_{\mathrm{w}} = \sin\theta_{\mathrm{W}}$ and $c_{\mathrm{w}} = \cos\theta_{\mathrm{W}}$ (θ_{W} is the Weinberg-Salam angle).

It is convenient to use generalized, effective Δq_i parameters of deviations for obtaining constraints. These parameters are determined the deviation of the differential cross section from a value in the SM in the process (2):

$$\begin{aligned} \Delta q_1\left(p_{\mathrm{eff}}^+, p_{\mathrm{eff}}^-\right) &= q_1^{\mathrm{SM+Z'}} - q_1^{\mathrm{SM}} = p_{\mathrm{eff}}^-\Delta q_{LR} + p_{\mathrm{eff}}^+\Delta q_{RL} , \\ \Delta q_2\left(p_{\mathrm{eff}}^+, p_{\mathrm{eff}}^-\right) &= q_2^{\mathrm{SM+Z'}} - q_2^{\mathrm{SM}} = p_{\mathrm{eff}}^-\Delta q_{LL} + p_{\mathrm{eff}}^+\Delta q_{RR} , \\ \Delta q_3\left(p_{\mathrm{eff}}^+, p_{\mathrm{eff}}^-\right) &= q_3^{\mathrm{SM+Z'}} - q_3^{\mathrm{SM}} . \end{aligned} \tag{7}$$

where $\Delta q_{\lambda_e,\lambda_f} = |q_{\lambda_e,\lambda_f}^{\mathrm{SM+Z'}}|^2 - |q_{\lambda_e,\lambda_f}^{\mathrm{SM}}|^2$.

3 Methodology for obtaining constraints

The methodology for obtaining the constraints on the effective parameters is based on the method of least squares. For further analysis, we assume that the results of future cross section measurements of process (2) are consistent with the SM predictions within the expected accuracy of measurements. In this case, the constraints on the $\mathbf{\Omega} = \Delta q_{1,2,3}$ parameters can be found using the criterion:

$$\chi^2(\mathbf{\Omega}) = \sum_{i=1}^{bins}\left[\frac{N_i^{\mathrm{SM+Z'}}(\mathbf{\Omega}) - N_i^{\mathrm{SM}}}{\delta N_i^{\mathrm{SM}}}\right]^2 \le \chi^2_{min} + \chi^2_{C.L.} , \tag{8}$$

where χ^2_{min} is defined by the minimum value requirement of the function $\chi^2(\mathbf{\Omega})$. For our case we can see that $\chi^2_{min} = 0$.

The value of the $\chi^2_{C.L.}$ function is set by the confidence level (C.L.). For the 95% confidence level by the quantile definition function $\chi^2_{C.L.} = 3.84, 5.99, 7.82$ can be found for the number of parameters equal to 1,2 and 3.

The experimental value is the number of events N_i^{SM} in the angular range $|z| \le 0.9$. The number of events $N_i^{\mathrm{SM+Z'}}(\mathbf{\Omega})$ induced by interactions containing the Z' we choose as a model function.

Assuming that the number of events in the bin follows the Poisson distribution and is relatively large (> 5), we have that the random error is equal to $\sqrt{N_i^{SM}}$. Considering the systematic error $\sim \delta_{syst}N_i^{SM}$, the error $\delta N_i^{SM} = \sqrt{N_i^{SM}(1 + \delta_{syst}^2 N_i^{SM})}$.

The number of events in the i-th bin is defined as:

$$N_i^{\mathrm{SM+Z'}} = \mathcal{L}_{int}\epsilon_f \int_{z_i}^{z_{i+1}} \left(\frac{d\sigma^{\mathrm{SM+Z'}}}{dz}\right) dz , \tag{9}$$

where $\mathcal{L}_{int}$ is the time-integrated luminosity and ϵ_f is the efficiency for $f\bar{f}$ reconstruction.

The algorithm for obtaining constraints includes three stages.

For the first step it is necessary to find the areas of change of parameters (7) for the different polarization observables, including the unpolarized case.

The second step is to obtain constraints on the $\Delta q_{\lambda_e,\lambda_f}$ parameters. In order to do this, it is necessary to consider two observables with different initial polarization $a = \{P_{e^-} = a_1, P_{e^+} = a_2\}$ and $b = \{P_{e^-} = b_1, P_{e^+} = b_2\}$. Using (8) for case a and b we can obtain the constraints, which are indicated by Δq_i^a and Δq_i^b respectively.

Using the system of equations (7) we can find:

$$\begin{aligned} \Delta q_{LR} &= \frac{p_{\mathrm{eff}}^{+,b}\Delta q_1^a - p_{\mathrm{eff}}^{+,a}\Delta q_1^b}{p_{\mathrm{eff}}^{-,a}p_{\mathrm{eff}}^{+,b} - p_{\mathrm{eff}}^{+,a}p_{\mathrm{eff}}^{-,b}} , \quad \Delta q_{RL} = \frac{p_{\mathrm{eff}}^{-,a}\Delta q_1^b - p_{\mathrm{eff}}^{-,b}\Delta q_1^a}{p_{\mathrm{eff}}^{-,a}p_{\mathrm{eff}}^{+,b} - p_{\mathrm{eff}}^{+,a}p_{\mathrm{eff}}^{-,b}} , \\ \Delta q_{LL} &= \frac{p_{\mathrm{eff}}^{+,b}\Delta q_2^a - p_{\mathrm{eff}}^{+,a}\Delta q_2^b}{p_{\mathrm{eff}}^{-,a}p_{\mathrm{eff}}^{+,b} - p_{\mathrm{eff}}^{+,a}p_{\mathrm{eff}}^{-,b}} , \quad \Delta q_{RR} = \frac{p_{\mathrm{eff}}^{-,a}\Delta q_2^b - p_{\mathrm{eff}}^{-,b}\Delta q_2^a}{p_{\mathrm{eff}}^{-,a}p_{\mathrm{eff}}^{+,b} - p_{\mathrm{eff}}^{+,a}p_{\mathrm{eff}}^{-,b}} , \end{aligned} \tag{10}$$

where values $p_{\mathrm{eff}}^{\pm,a}$ and $p_{\mathrm{eff}}^{\pm,b}$ are calculated for a set of polarizations a and b.

The goal of the third step is to find the range of change of the physical parameters of Z' ($g_{\mathrm{Z}',f}^{L,R}$, $m_{\mathrm{Z}'}$ and $\Gamma_{\mathrm{Z}'}$) using explicit form of the function $\Delta q_{\lambda_e,\lambda_f}$.

4 Constraints on Z' physical parameters

As an example of the proposed technique, we can find the constraints for the process $e^+e^- \to \tau\bar{\tau}$ at the ILC energy with parameters: $\sqrt{s} = 1$ TeV, $\mathcal{L}_{int} = 8$ ab^{-1}, $\epsilon_\tau = 65\%$ and $\delta_{syst} = 0.5\%$. It is assumed that, for the Z' the universality among the family of lepton couplings is fulfilled $g_{\mathrm{Z}',e}^{\rho} = g_{\mathrm{Z}',\mu}^{\rho} = g_{\mathrm{Z}',\tau}^{\rho}$. In the presence of initial polarization, a 20% decrease luminosity must be taken into account. The one-dimensional constraints on the $\Delta q_{1,2,3}$ parameters are presented in table (1).

Table 1: The one-dimensional model-independent constraints on the $\Delta q_{1,2,3}$ $(C.L. = 68.27\%)$

P_{e^-}/P_{e^+}	$\Delta q_1 \times 10^{-3}$	$\Delta q_2 \times 10^{-3}$	Δq_3
$0/0$	∓ 4.27	∓ 8.85	∓ 715
$0.8/-0.2$	∓ 4.33	∓ 8.44	∓ 710
$-0.8/0.2$	∓ 4.46	∓ 9.63	∓ 757

Constraints on the Δq_3 are anomalously large because the corresponding addend in the cross section (3) is proportional to the $\eta_f^2 = 4m_f^2/s$. The value of $\eta_f^2 = 1.26289 \times 10^{-5}$ for $\sqrt{s} = 1$ TeV and $m_\tau = 1.77686$ is easy to find. Therefore, the value of the Δq_3 is large within the $\chi^2(\Delta q_3) \le 1$. Obviously, if we use the obtained constraints on the Δq_3 we do not improve the physical constraints on Z'.

Figure 1(a) shows the two-dimensional constraints on the Δq_1 and Δq_2 for the 3 values of confidence level. The area with $C.L. = 39.35\%$ gives the one-dimensional constraints, where are the Δq_1 and Δq_2 independently of one another in the confidence range with $C.L. = 68.27\%$.

Considering correlation between the parameters $\Delta q_{1,2,3}$, it is possible to get the constraints in the form of an ellipsoid. The three-dimensional constraints on the $\Delta q_{1,2,3}$ are obtained for $C.L. = 39.35\%$ and presented on Figure 1(b).

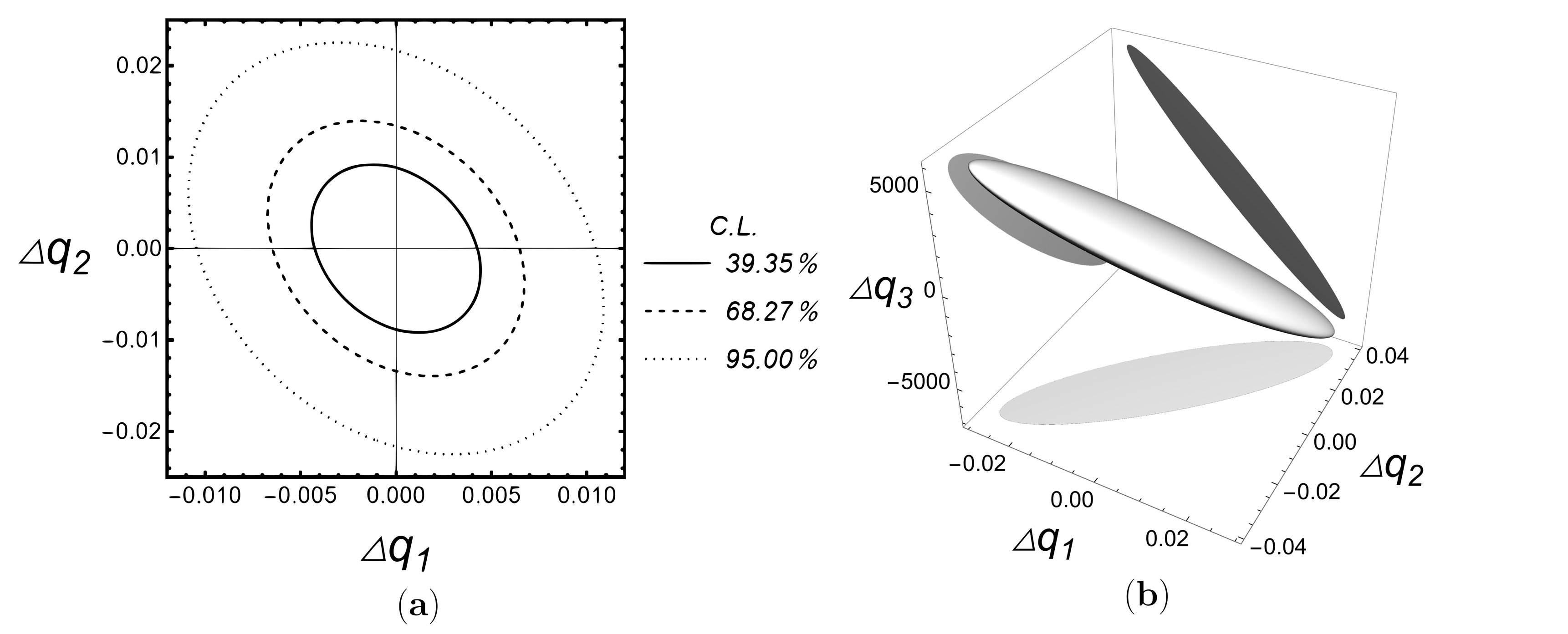

Figure 1: Two-dimensional (a) and three-dimensional (b) model-independent constraints on effective parameters Δq_i in the process $e^+e^- \to \tau\bar{\tau}$ for unpolarized initial beams

Using the values from table (1) and the equations (10) with $a = \{P_{e^-} = 0.8, P_{e^+} = -0.2\}$ and $b = \{P_{e^-} = -0.8, P_{e^+} = 0.2\}$, we can find:

$$\begin{aligned} -0.0022 \le \Delta q_{LR}\ ,\Delta q_{RL} \le 0.0022\ , \\ -0.0049 \le \Delta q_{LL} \le 0.0049\ , \\ -0.0042 \le \Delta q_{RR} \le 0.0042\ . \end{aligned} \tag{11}$$

After this, using the equations (5) we need to identify the $\Delta q_{\lambda_e,\lambda_f}$ as functions of Z' physical parameters. Assuming that $\Gamma_{\mathrm{Z}'} = 0.1 \times m_{\mathrm{Z}'}$ we can find the constraints on the $m_{\mathrm{Z}'}$ and $g_{\mathrm{Z}',l}^{\lambda_l} \times g_{\mathrm{Z}',l}^{\lambda_l}$ (Figure 2).

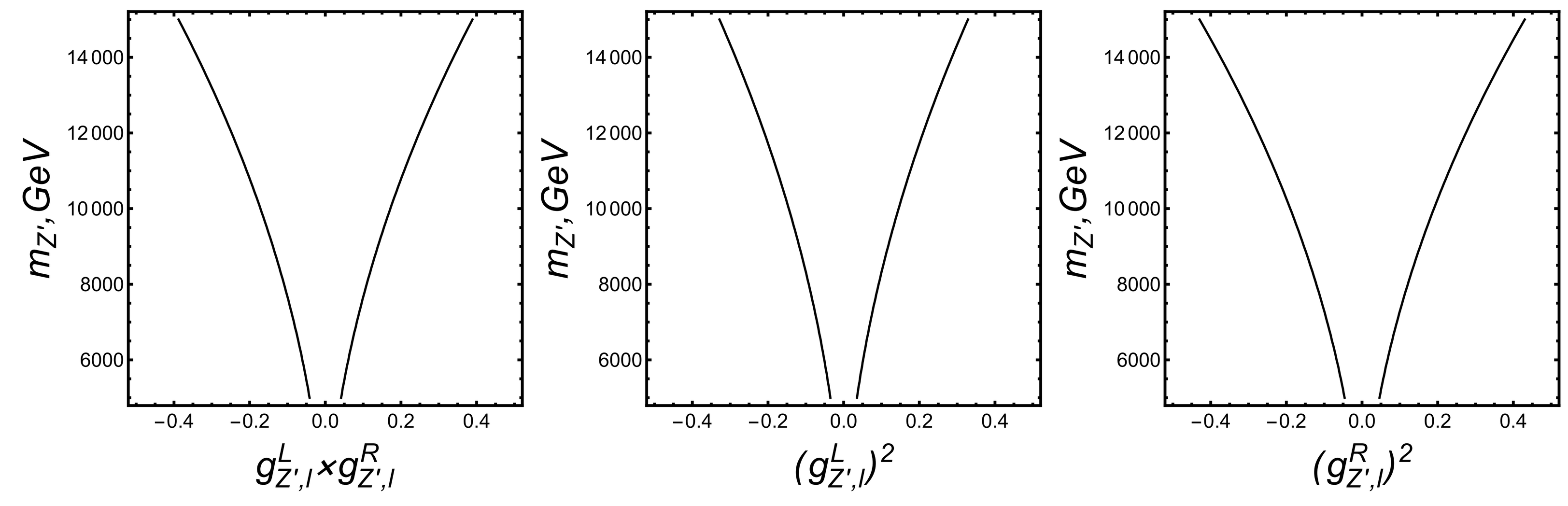

Figure 2: Constraints on the $m_{Z'}$ and $g_{\mathrm{Z}',l}^{\lambda_l} \times g_{\mathrm{Z}',l}^{\lambda_l}$ in the process $e^+e^- \to \tau\bar{\tau}$

5 Conclusion

This paper proposes a methodology for obtaining constraints on Z' physical parameters in the process (2). The technique is based on the representation of the differential cross section with three real effective parameters. Constraints on the physical parameters ($m_{\mathrm{Z}'}$ and $g_{\mathrm{Z}',l}^{\lambda_l} \times g_{\mathrm{Z}',l}^{\lambda_l}$) for the ILC experiment at energy of 1 TeV are obtained.

For further research, it is interesting to expand the number of observables (A_{FB}, A_{LR} and other) and perform a model-dependent analysis, as well as take into account the impact of radiative corrections.

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