Cosmological Constant Suppression in Non-Stationary Scalar Covariant State

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Introduction

- Cosmological constant problem: according to the data — 10^{-3} eV, based on estimates in theoretical models — 10^{18} GeV;
- No theoretical models with vacuum energy density suppression $\Rightarrow$ Zeldovich: the cosmological constant $\leftrightarrow$ density of vacuum energy generated by ZPM;
- 4D isotopic model in contrast to 3D isotropic

Zero-point modes of scalar field and spatial covariance

- For SE tensor in ultra-relativistic expressions ($\Lambda_P \gg m$):

$$\langle 0|T_{\alpha\beta}|0\rangle \approx \frac{1}{3}\,\delta_{\alpha\beta}\int\limits_0^{\Lambda_P}\frac{4\pi p^2 \mathrm{d}p}{(2\pi)^3}\,\frac{p^2}{2p}\Big(1-\frac{m^2}{2p^2}\Big) = \frac{1}{3}\,\delta_{\alpha\beta}\cdot\frac{\Lambda_P^4}{16\pi^2}\Big(1-\frac{m^2}{\Lambda_P^2}\Big),$$

$$\langle 0|T_{00}|0\rangle \approx \int\limits_0^{\Lambda_P}\frac{4\pi p^2 \mathrm{d}p}{(2\pi)^3}\,\frac{p}{2}\Big(1+\frac{m^2}{2p^2}\Big) = \frac{\Lambda_P^4}{16\pi^2}\Big(1+\frac{m^2}{\Lambda_P^2}\Big)\,.$$

- Therefore, the ratio of isotropic pressure:

$$w = \frac{\mathrm{p}}{\rho} \approx \frac{1}{3}\Big(1-\frac{2}{m^2}\Lambda_P^2\Big) \to w_{\mathrm{rad}} = \frac{1}{3} \quad \text{at } \frac{m}{\Lambda_P} \to 0.$$

- Analogously, in the non-relativistic limit:

$$w = \frac{\mathrm{p}}{\rho} \approx \frac{3}{5}\,\frac{\Lambda_P^2}{m^2}\Big(1-\frac{23}{35}\,\frac{\Lambda_P^2}{m^2}\Big) \to w_{\mathrm{m}} = 0 \quad \text{at } \frac{\Lambda_P}{m} \to 0.$$

Scalar field and spatial-temporal covariance

- Suppose true spatial-temporal structure (Wick rotation, no poles):

$$\langle \text{vac} \,|\, T_{\mu\nu}(x) \,|\, \text{vac} \rangle = \int \frac{d^4 p_E}{(2\pi)^4} \frac{1}{p_E^2 + m^2} \left(p_\mu^E p_\nu^E - \frac{1}{2} g_{\mu\nu}^E (p_E^2 + m^2) \right)$$

- Replacement:

$$p_\mu^E p_\nu^E \mapsto \frac{1}{4} g_{\mu\nu}^E p_E^2 \,.$$

- Therefore:

$$\langle \text{vac} \,|\, T_{\mu\nu}(x) \,|\, \text{vac} \rangle = \frac{1}{4} g_{\mu\nu} \int \frac{d^4 p_E}{(2\pi)^4} \left(1 + \frac{m^2}{p_E^2 + m^2} \right) \Rightarrow w_{\text{vac}} = -1$$

- True structure of spatial-temporal quantum fluctuations in the vacuum supposes the covariant four-dimensional formulation.

Finite volume and spatial structure

- The scalar field in a finite volume:

$$S = V_{[3]} \int \mathrm{d}t \, \frac{1}{2} \left((\partial_t \phi(t))^2 - m^2 \phi^2(t) \right)$$

- Using oscillator coordinate, mass, frequency and scale of energy:

$$\langle 0 | m^2 \phi^2 | 0 \rangle = \langle 0 | (\partial_t \phi)^2 | 0 \rangle = \frac{1}{2} \frac{m}{V_{[3]}},$$

- So, the energy density equals:

$$\langle 0 | T_{00} | 0 \rangle = \frac{1}{2} \langle 0 | (\partial_t \phi)^2 + m^2 \phi^2 | 0 \rangle = \frac{1}{2} \frac{m}{V_{[3]}},$$

- while the pressure is given by:

$$\mathtt{p} = \frac{1}{2} \langle 0 | (\partial_t \phi)^2 - m^2 \phi^2 | 0 \rangle = 0.$$

- Does not correspond to the vacuum: the parameter fits the **dust**, i.e. particles with zeroth pressure

Covariant model

- Action for the field in the **finite** volume in the **covariant 4d** form:

$$S_\Lambda = \frac{1}{\Lambda^3}\int \mathrm{d}\tau \frac{1}{2}\Big((\partial_\tau\phi)^2\big\langle \partial_\mu\tau\partial_\nu\tau\big\rangle g^{\mu\nu} - m^2\phi^2\Big),$$

- the covariant ST-structure in the model is defined by:

$$\big\langle \partial_\mu\tau\partial_\nu\tau\big\rangle = \frac{1}{4}g_{\mu\nu}, \quad \langle(\partial_\lambda\tau)^2\rangle = 1$$

- Analogously for the basic state with no quanta of oscillator:

$$\langle \mathrm{vac}\,|\,T^\Lambda_{\mu\nu}\,|\,\mathrm{vac}\rangle = g_{\mu\nu}\langle \mathrm{vac}\,|\,\big(-\tfrac{1}{4}(\partial_\tau\phi)^2 + \tfrac{1}{2}m^2\phi^2\big)\,|\,\mathrm{vac}\rangle = g_{\mu\nu}\frac{1}{8}\,m\Lambda^3\,.$$

- and the bare value of energy density:

$$\rho^{\mathrm{bare}}_{\mathrm{vac}} = \langle \mathrm{vac}\,|\,T^\Lambda_{00}\,|\,\mathrm{vac}\rangle = \frac{1}{8}\,m\Lambda^3 > 0.$$

Non-stationary coherent state

- The contribution of vacuum state to the average stress-energy tensor is suppressed as:

$$\langle T^{\Lambda}_{\mu\nu}\rangle_{\rm vac} = \langle \mathsf{vac}\,|\,T^{\Lambda}_{\mu\nu}\,|\,\mathsf{vac}\rangle \cdot \mathrm{e}^{-\langle n\rangle} = \rho^{\rm bare}_{\rm vac}\cdot \mathrm{e}^{-\langle n\rangle} = \frac{1}{8}\,m\Lambda^3\cdot \mathrm{e}^{-\langle n\rangle},$$

- the energy of coherent state in the final volume:

$$\langle E\rangle = \omega_{\rm osc}\big(\langle n\rangle + \tfrac{1}{2}\big) = m\big(\langle n\rangle + \tfrac{1}{2}\big),$$

- the same quantity in terms of energy density within a finite volume:

$$\langle E\rangle = V_{[3]}\,\rho^{\rm bare}_{\rm vac}\big(\langle n\rangle + \tfrac{1}{2}\big)\cdot 2 = V_{[3]}\,\frac{1}{4}\,m\Lambda^3(\langle n\rangle + \tfrac{1}{2}\big),$$

- Therefore, the reference volume:

$$V_{[3]} = \frac{4}{\Lambda^3}$$

Numerical estimates

- The contribution of vacuum state to the average stress-energy tensor is suppressed as:

$$\rho_{\rm vac} \sim (10^{-3}\ \mathrm{eV})^4, \qquad \Lambda \sim 10^{16}\ \mathrm{GeV}\,.$$

- Then $\langle n \rangle \sim 250 \Rightarrow$ the energy of initial coherent state in the reference volume:

$$\langle E \rangle \sim \langle n \rangle m \sim \tilde{m}_{\rm Pl}\,.$$

- Two energy scales the reduced Planckean mass and the scale of inflation plateau.

Conclusion

- We have shown the bare value of ZP can be suppressed under the following conditions (+covariant formalism):
  - the non-stationary coherent state of scalar field in the expanding universe
  - the finite volume
  - natural estimates for the cut-off scale
- The model involves at least two scales of energy

Thank you for your attention!