Cosmology and particle physics
Lecture #3
Hot Big Bang and Dark Matter models

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Outline

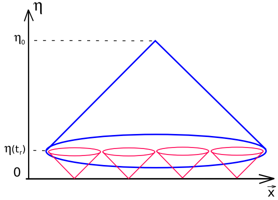
Cosmological (particle) horizon $l_H(t)$

distance covered by photons emitted at $t = 0$

the size of causally-connected region — the size of the visible part of the Universe

in conformal coordinates:
$ds^2 = 0 \longrightarrow |d\mathbf{x}| = d\eta$
coordinate size of the horizon equals
$\eta(t) = \int d\eta$

$$l_H(t) = a(t)\eta(t) = a(t)\int_0^t \frac{dt'}{a(t')}$$



dust

$$l_H(t) = 3t = \frac{2}{H(t)}\,, \quad l_{H,0} = 2.6\times 10^{28}\text{cm} \quad (h = 0.7)$$

Recombination: horizon

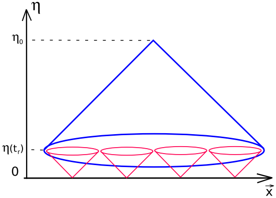
matter domination: $l_{\mathrm{H},r} = 2H_r^{-1}$

$$H_r^2 = \frac{8\pi}{3} G\rho_{\mathrm{M}}(t_r) = \frac{8\pi}{3} G\rho_{\mathrm{M},0}\left(\frac{a_0}{a_r}\right)^3 = \frac{8\pi}{3} G\rho_c \Omega_{\mathrm{M},0}(1+z_r)^3 \, .$$

at recombination: $l_{\mathrm{H},r} = \frac{2}{H_0\sqrt{\Omega_{\mathrm{M}}}}\frac{1}{(1+z_r)^{3/2}}$

today: $l_{\mathrm{H},r}(t_0) = l_{\mathrm{H},r} \times \frac{a_0}{a_r} = \frac{2}{H_0\sqrt{\Omega_{\mathrm{M}}}}\frac{1}{\sqrt{1+z_r}}$

$$\frac{l_{H_0}}{l_{\mathrm{H},r}(t_0)} \sim \sqrt{1+z_r} \simeq 30$$



Examples of cosmological solutions

radiation: $p = \frac{1}{3}\rho$ singular at $t = t_s$

$$\rho = \frac{\text{const}}{a^4}\,, \quad a(t) = const \cdot (t - t_s)^{1/2}\,, \quad \rho(t) = \frac{\text{const}}{(t-t_s)^2}$$

$$t_s = 0\,, \quad H(t) = \frac{\dot{a}}{a}(t) = \frac{1}{2t}\,, \quad \rho = \frac{3}{8\pi G}H^2 = \frac{3}{32\pi G}\frac{1}{t^2}$$

$$l_H(t) = a(t)\int_0^t \frac{dt'}{a(t')} = 2t = \frac{1}{H(t)}\,.$$

In case of thermal equilibrium $T = \text{const}/a$

$$\rho_b = \frac{\pi^2}{30}g_b T^4\,, \quad \rho_f = \frac{7}{8}\frac{\pi^2}{30}g_f T^4$$

$$\rho = \frac{\pi^2}{30}g_* T^4\,, \qquad g_* = \sum_b g_b + \frac{7}{8}\sum_f g_f = g_*(T)$$

$ds^2 = dt^2 - e^{2H_{dS}t} d\mathbf{x}^2$

no cosmological horizon: $l_H(t) = e^{H_{dS}t} \int_{-\infty}^{t} dt' e^{-H_{dS}t'} = \infty$

de Sitter (events) horizon ($\mathbf{x} = 0$, t):

from which distance $l(t)$ one can detect light emitted at t?

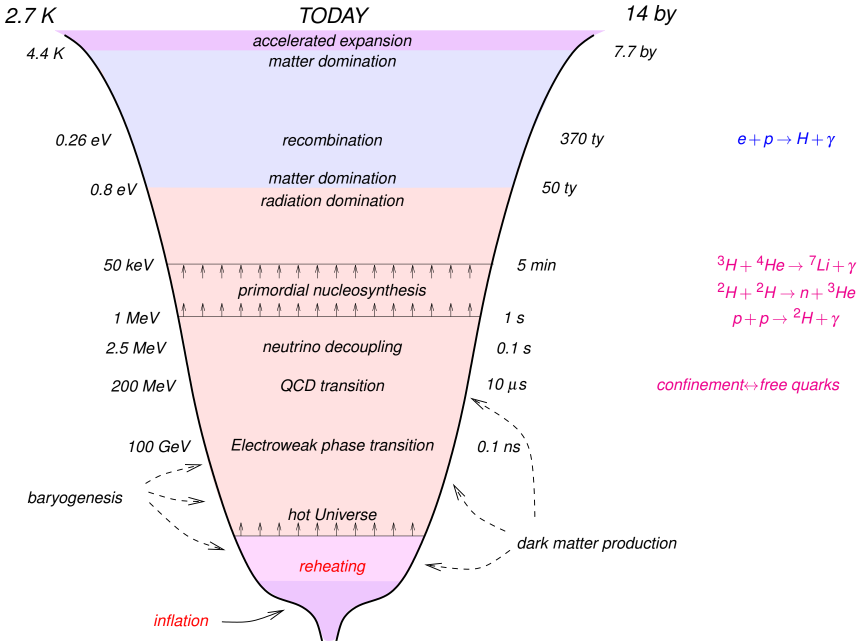
in conformal coordinates: $ds^2 = 0 \longrightarrow |d\mathbf{x}| = d\eta$

coordinate size: $\eta(t \to \infty) - \eta(t) = \int_t^\infty \frac{dt'}{a(t')}$

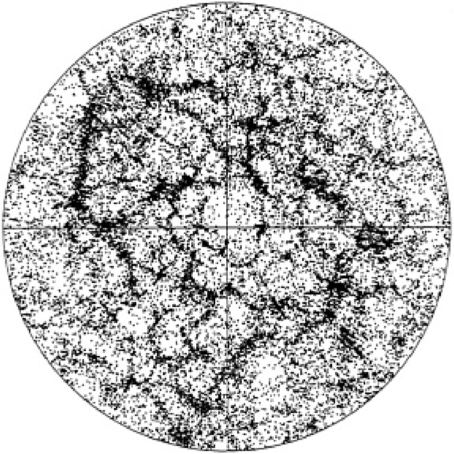
physical size: $l_{dS} = a(t) \int_t^\infty \frac{dt'}{a(t')} = \frac{1}{H_{dS}}$

observer will never be informed what happens at distances larger than $l_{dS} = H_{dS}^{-1}$

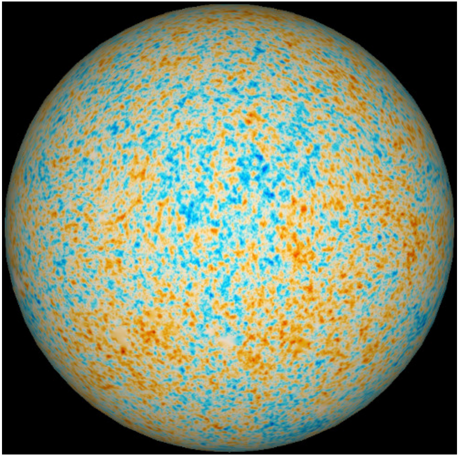
Our future? with $H_{dS} = 0.8 \times H_0$


2.7 K
TODAY
14 by
accelerated expansion
4.4 K
matter domination
7.7 by
0.26 eV
recombination
370 ty
$e + p \to H + \gamma$
matter domination
0.8 eV
radiation domination
50 ty
50 keV
5 min
$^3H + ^4He \to ^7Li + \gamma$
primordial nucleosynthesis
$^2H + ^2H \to n + ^3He$
1 MeV
1 s
$p + p \to ^2H + \gamma$
2.5 MeV
neutrino decoupling
0.1 s
200 MeV
QCD transition
10 μs
confinement↔free quarks
100 GeV
Electroweak phase transition
0.1 ns
baryogenesis
hot Universe
dark matter production
reheating
inflation

Recombination: $p + e \to H + \gamma$, $T_{rec} \approx 0.25\,\text{eV}$



Large Scale Structure



CMB anisotropy

Small inhomogeneities in the expanding Universe

matter perturbations (perfect fluid approximation)

$$T^0_0 \to \rho(t) + \delta\rho(\eta, \mathbf{x}), \quad T^0_i \to \partial_i v(\eta, \mathbf{x}), \quad T^i_j \to \delta p(\eta, \mathbf{x})$$

gravitational perturbations (scalar and tensor modes)

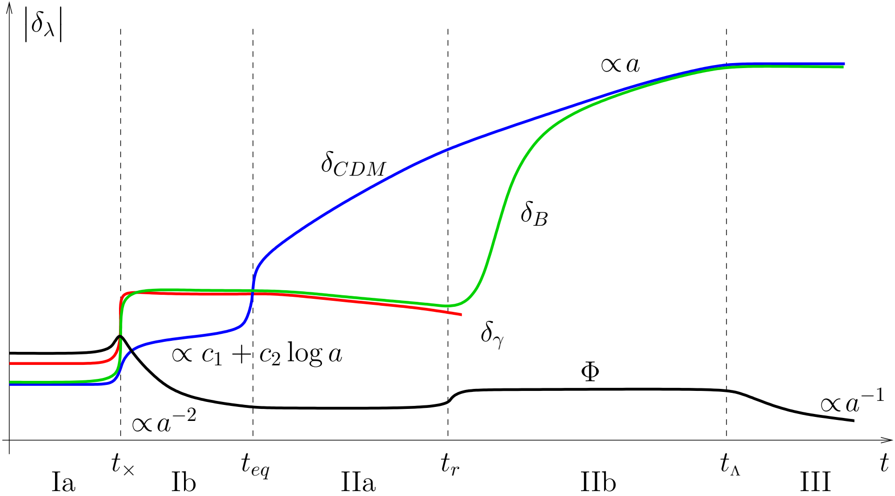
$$ds^2 = a^2(\eta)\left[(1+2\Phi(\eta,\mathbf{x}))\,d\eta^2 - (1+2\Psi(\eta,\mathbf{x}))\,\mathbf{dx}^2 - h^{TT}_{ij}(\eta,\mathbf{x})dx^i dx^j\right]$$

Equations for linear perturbations, $\delta\rho/\rho \equiv \delta \ll 1$, $\Phi \ll 1$, etc

$$R_{\mu\nu} + \frac{1}{2}Rg_{\mu\nu} = 8\pi G T_{\mu\nu} \quad \to \quad \dots$$

$$\nabla_\mu T^{\mu\nu} = 0 \quad \to \quad \dots$$

Subhorizon modes ($k/a > H$) at various stages



On formulas. . .

- short waves, $k\eta_{eq} \gg 1$ $\qquad R_B \equiv 3\rho_B/4\rho_\gamma$

$$\delta_\gamma = \Phi_{(i)} \cdot \left[-324 \cdot (1+R_B)\, l^2(\Omega_M)\, \frac{\Omega_{CDM}}{\Omega_M}\, (1+z_{eq})\, \frac{\log(0.2k\eta_{eq})}{(k\eta_0)^2} + \frac{6}{(1+R_B)^{1/4}} \cos\left(k \int_0^\eta d\tilde{\eta}\, u_s \right) \right] ,$$

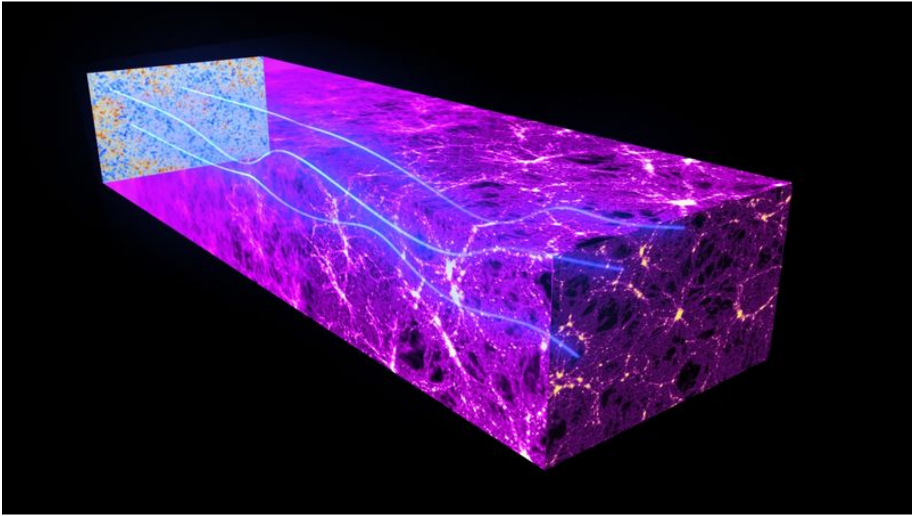
- long waves, $k\eta_{rec} \ll 1$

$$\delta_\gamma = -\frac{12}{5}\Phi_{(i)} = \text{const}$$

- intermediate waves ...

$$\delta_\gamma(\mathbf{k},\eta) = -4\left[1+R_B(\eta)\right]\Phi(\mathbf{k},\eta) + 4\Phi_{(i)}(\mathbf{k}) \cdot A(k,\eta) \cos\left(k\int_0^\eta u_s d\tilde{\eta} \right) ,$$

On top of that: propagation in expanding Universe



On formulas...

From linear approximation to the geodesic equation...
for scalar perturbations

$$\frac{\delta T}{T}(\mathbf{n},\eta_0) = \frac{1}{4}\delta_\gamma(\eta_r) + (\Phi(\eta_r) - \Phi(\eta_0)) + \int_{\eta_r}^{\eta_0}\left(\Phi' - \Psi'\right) d\eta + \mathbf{nv}(\eta_r) - \mathbf{nv}(\eta_0)\,.$$

for tensor perturbations

$$\frac{\delta T}{T}(\mathbf{n},\eta_0) = \frac{1}{2}\int_{\eta_r}^{\eta_0} d\eta\, n_i h_{ij}^{TT\,\prime} n_j\,,$$

These inhomogeneities (matter perturbations)

originate from the initial matter density (scalar) perturbations

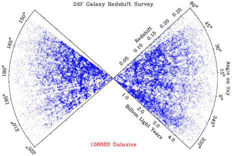
$\delta\rho/\rho \sim \delta T/T \sim 10^{-4}$, which are

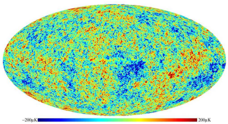
adiabatic $\qquad \delta\left(\frac{n_B}{s}\right) = \delta\left(\frac{n_{DM}}{s}\right) = \delta\left(\frac{n_L}{s}\right)$

Gaussian $\qquad \langle \frac{\delta\rho}{\rho}(\mathbf{k}) \frac{\delta\rho}{\rho}(\mathbf{k}') \rangle \propto \left(\frac{\delta\rho}{\rho}(\mathbf{k})\right)^2 \times \delta(\mathbf{k}+\mathbf{k}')$

flat spectrum $\qquad \langle \left(\frac{\delta\rho}{\rho}(\mathbf{x})\right)^2 \rangle = \int_0^\infty \frac{\mathbf{dk}}{\mathbf{k}} \mathscr{P}_S(\mathbf{k}) \qquad \mathscr{P}_S(\mathbf{k}) \approx \mathrm{const}$

LSS and CMB $\qquad \mathscr{P}_S \equiv A_S \times \left(\frac{k}{k_*}\right)^{n_s - 1} \qquad A_S \approx 2.5 \times 10^{-9}\,, \quad n_S \approx 0.97$





Standard cosmological model $ds^2 = dt^2 - a^2(t)dx^2$

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv H^2 = H_0^2 \left[\Omega_\Lambda + (\Omega_{DM} + \Omega_B + \Omega_{\nu,m\neq 0})\left(\frac{a_0}{a}\right)^3 + (\Omega_\gamma + \Omega_{\nu,m=0})\left(\frac{a_0}{a}\right)^4\right]$$

- $T_\gamma = 2.735\,\mathrm{K}, \quad \Longrightarrow \quad \Omega_\gamma \sim 10^{-5}$
- $N_\nu \approx 3, \quad \sum m_\nu < 0.2\,\mathrm{eV} \quad \Longrightarrow \quad \Omega_{\nu,\neq 0}, \Omega_{\nu,0} \sim 10^{-5}$?
- $\Omega_B = 4.5\% \quad \Longrightarrow \quad \eta_B \equiv n_B/n_\gamma = 6\times 10^{-10}$
- $\Omega_{DM} = 27.5\%$
- $H_0 = 67\,\mathrm{km/s/Mpc} \quad \Longrightarrow \quad \rho_0 = 5\,\mathrm{GeV/m^3}$
- $\Omega_\Lambda = 68\% \quad \Longrightarrow$ flat space
- adiabatic, gaussian matter perturbations

$$\langle\left(\frac{\delta\rho}{\rho}\right)^2\rangle \sim A_S \int \frac{dk}{k}\left(\frac{k}{k_*}\right)^{n_S-1}$$

  with $A_S = 3\times 10^{-9}$ and $n_S = 0.97$
- no tensor perturbations, $r \equiv A_T/A_S < 0.05$
- reionization at $z \equiv a_0/a = 10$

Friedmann equation $\quad (00): \quad \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3} G\rho - \frac{\varkappa}{a^2}$

$$\nabla_\mu T^{\mu 0} = 0 \quad \longrightarrow \quad \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$$

the equation of state

$$p = p(\rho)$$

many-component liquid, in case of thermal equilibrium

other equations

$$-3d(\ln a) = \frac{d\rho}{p+\rho} = d(\ln s)$$

entropy of cosmic primordial plasma is conserved in a comoving frame

$$sa^3 = \text{const}$$

Dark Matter: many well-motivated candidates

- WIMPs — related to EW scale, SUSY
- sterile neutrinos — active neutrino oscillations
- light scalar field — string theory
- axion — strong CP-problem
- gravitino — local SUSY
- Heavy relics — GUTs
- (Topological) defects — GUTs
- Massive Astrophysical Compact Heavy Objects
- Primordial black hole (remnants) — Phase transitions, exotic inflation, reheating

Multicomponent Dark Matter ?

γ, ν, H, He

Microscopic processes in the expanding Universe

A competition between scattering, decays, etc and expansion

for general processes one should solve kinetic equations

$$\frac{dn_{X_i}}{dt} + 3Hn_{X_i} = \sum(production - destruction)$$

Boltzmann equation in a comoving volume: $\frac{d}{dt}\left(n\,a^3\right) = a^3 \int \ldots$

production:

$$\sigma(A+B \to X+C)n_A n_B, \quad \Gamma(D \to E+X)n_D \cdot M_D/E_D, \quad \text{etc}$$

desrtuction:

$$\sigma(A+X \to C+B)n_A n_X, \quad \Gamma(X \to F+G)n_X \cdot M_X/E_X, \quad \text{etc}$$

Fast direct and inverse processes, $\Gamma \gtrsim H$, are in equilibrium,

$\sum(\;) = 0$ and thermalize particles

Freeze-out of nonrelativistic Dark Matter

Assumptions:

1. no $X-\bar{X}$ asymmetry — either $X=\bar{X}$ or $n_X = n_{\bar{X}}$
2. @ $T \lesssim M_X$ in thermal equilibrium with plasma — (e.g. neutrons)

$$n_X = n_{\bar{X}} = g_X \left(\frac{M_X T}{2\pi}\right)^{3/2} \mathrm{e}^{-M_X/T}$$

$$X\bar{X} \longrightarrow \text{light particles}$$

freeze-out temperature T_f $\qquad H \equiv T^2/M_{Pl}^*, \quad M_{Pl}^* = M_{Pl}/1.66\sqrt{g_*}$

$$n_X \langle \sigma_{ann} v \rangle = H(T_f) \longrightarrow T_f = \frac{M_X}{\ln\left(\frac{g_X M_X M_{Pl}^* \sigma_0}{(2\pi)^{3/2}}\right)}\;.$$

Bethe formula: $\qquad$ s-wave: $\sigma_{ann} = \frac{\sigma_0}{v}$

Weakly Interacting Massive Particles

density after freeze-out: $n_X(T_f) = \frac{T_f^2}{M_{Pl}^* \sigma_0}$

present density: $n_X(T_0) = \left(\frac{a(T_f)}{a(T_0)}\right)^3 n_X(T_f) = \left(\frac{s_0}{s(T_f)}\right) n_X(T_f) \propto \frac{1}{T_f}$

$X + \bar{X}$ contribution to critical density:

$$\Omega_X = 2\frac{M_X n_X(T_0)}{\rho_c} = 7.6 \frac{s_0 \ln\left(\frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}}\right)}{\rho_c \sigma_0 M_{Pl} \sqrt{g_*(T_f)}}$$

$$= 0.1 \cdot \left(\frac{(10\ \text{TeV})^{-2}}{\sigma_0}\right) \frac{10}{\sqrt{g_*(T_f)}} \ln\left(\frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}}\right) \cdot \frac{1}{2h^2}$$

WIMPs: discussion

$$\Omega_X = 0.1 \cdot \left(\frac{(10\ \text{TeV})^{-2}}{\sigma_0} \right) \frac{10}{\sqrt{g_*(T_f)}} \ln \left(\frac{g_X M_{Pl}^* M_X \sigma_0}{(2\pi)^{3/2}} \right) \cdot \frac{1}{2h^2}$$

- natural DM: subweak-scale cross section $\sigma_0 \sim 0.01 \times \sigma_W$
  say, $M_X \sim 1$ TeV or X is not a weak gauge eigenstate
- naturaly “light” unitarity $\sigma_0 \lesssim \frac{4\pi}{M_X^2} \longrightarrow M_X \lesssim 100$ TeV
- all stable particles with smaller σ_0 are forbidden !!
- WIMPs remain in kinetic equilibrium with plasma till $T \sim 10$ MeV

  this is Cold Dark Matter, $v_{RD/MD} \ll 10^{-3}$

  WIMPs may form dark halos (clumps) much lighter than dwarf galaxies

Weakly IMPs are mostly welcome (e.g. LSP in SUSY)

We can fully explore the model !!

- Direct searches for Galactic Dark Matter ($v \sim 10^{-3}$) a hit

$$X + \text{nuclei} \to X + \text{nuclei} + \Delta E$$

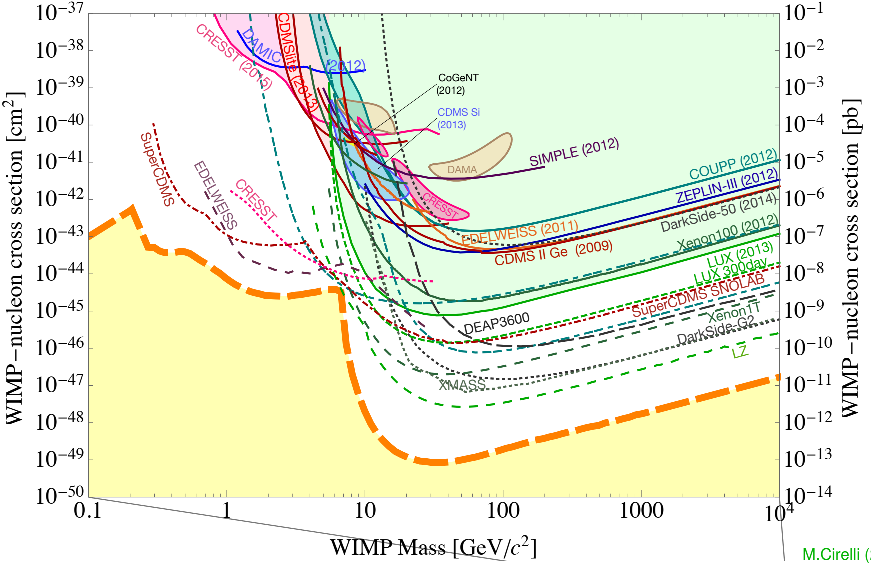
- Can search for WIMPs in cosmic rays: products of WIMPs annihilation (in Galactic center, dwarf galaxies, Sun) $\propto n^2$

$$X + \bar{X} \to p\bar{p}\,,\;\; e^+e^-\,,\;\; \nu, \gamma, \ldots$$

- Can search for WIMPs in collision experiments (LHC): missing

$$X + \bar{X} \leftrightarrow \text{SM} + \text{SM'} + \ldots$$

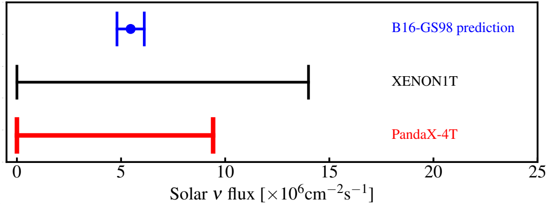
Prospects in WIMP searches

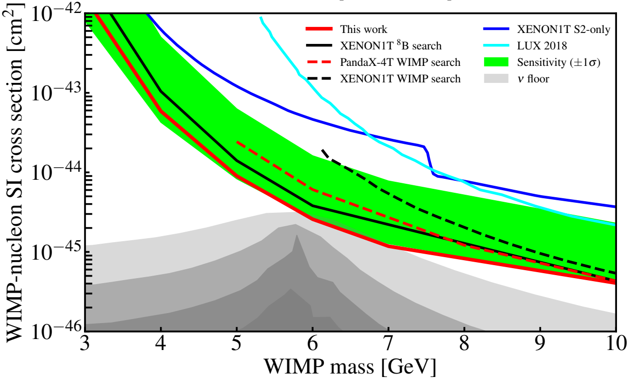


M.Cirelli (2015)

Testing neutrino floor with PandaX-4T

2207.04883



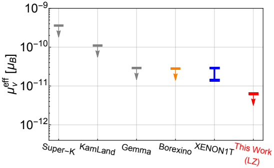


Other by-product results:

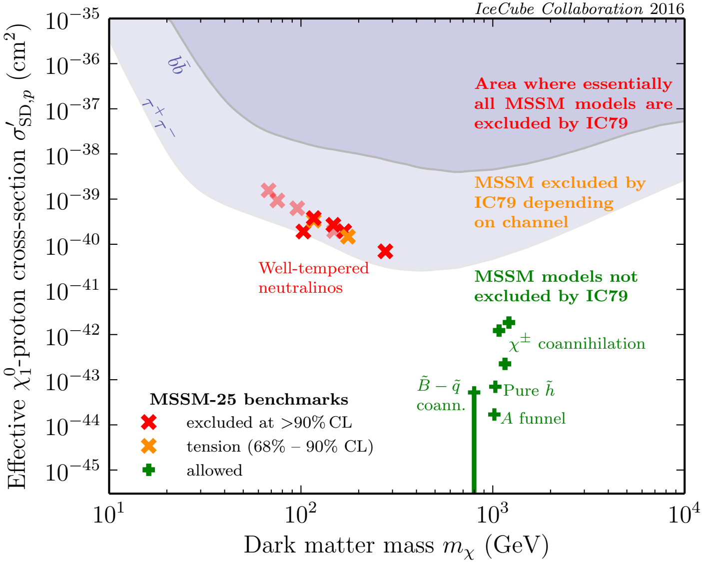
2207.05036

the strongest limit

on μ_ν from LZ:

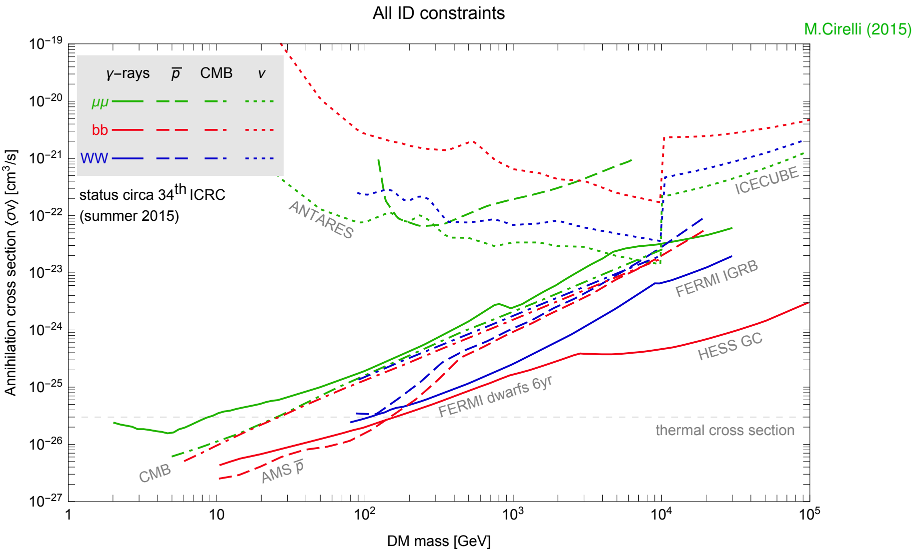


Constraining the DM model parameter space



M.G. Aartsen at al (2016)

Present indirect limits on DM annihilation (clumps..)



Next generation: CTA

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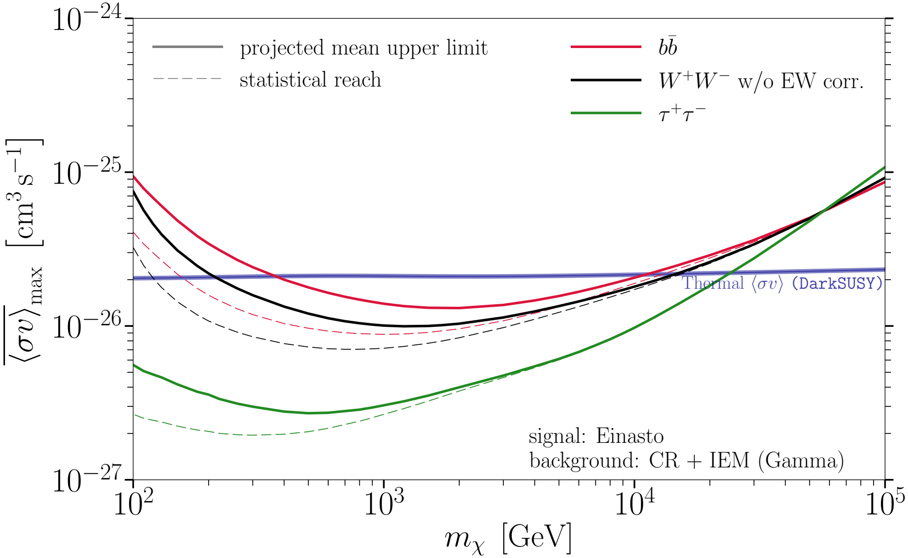
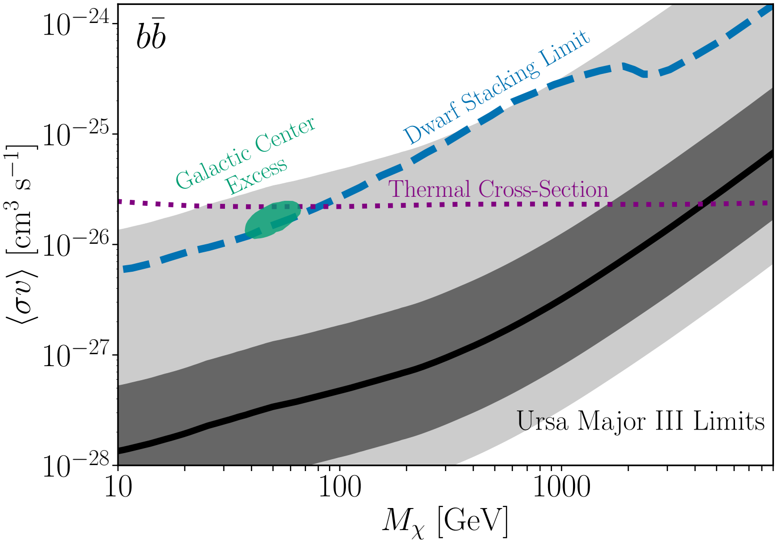


Ursa Major III

2311.14611



Ursa Major III

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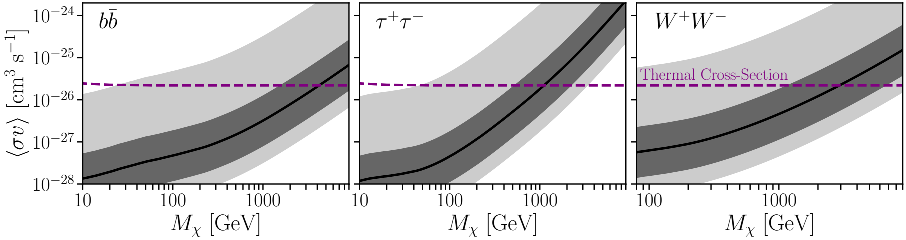
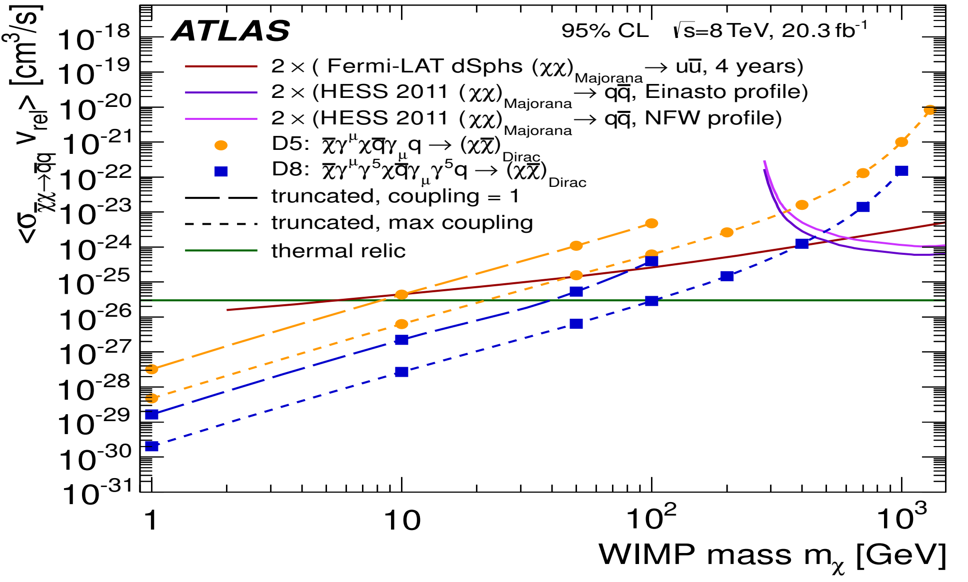


LHC limits for annihilation

1502.01518



If thermal CDM but not Weakly IMPs?

We still can study the model if DM annihilates (partly) into SM particles

- But DM particle X can be light and feebly coupled (t-channel)

$$\sigma_0 \sim \frac{\xi^4}{M_X^2}$$

ξ is not a gauge coupling within GUT !

- With small σ_0 one needs entropy production
- σ_0 may be increased by s-channel resonance, $M_Y \approx 2M_X$
- annihilation can be amplified by co-annihilation channels, $X + A \to SM$
- With light messengers between Dark and Visible sectors many estimates change, say $\sigma_0 = \sigma_0(v)$
- DM interaction at freeze-out and now are not the same
  say, Sommerfield enhancement of the annihilation of slow particles $v \sim 10^{-3}$

Dark Matter: non-thermal production

1. in the primordial plasma of SM particles
   (via scatterings (freeze-in), — gravitino
   via oscillations): — sterile neutrino of 1-50 keV
2. at phase transitions:
   - axion of $10^{-4} - 10^{-7}$ eV
   - Q-balls
   - strangelets (?)
3. during reheating (after inflation?): — black holes
   - any guy coupled (only) to inflaton
   - perturbatively: — inflaton decays
     - production by external (inflaton) field
   - non-perturbatively: — Bose-enhancement of
     - coherent production by external field
4. while the Universe expands:
   - gravity produces any particles at $H \sim M_X$

A simple example of scalar DM

most general renormalizable coupled to SM:

Z_2-invariant Higgs (Φ) portal

$$\Delta\mathscr{L} = \frac{1}{2}g^{\mu\nu}\partial_\mu X\partial_\nu X - \frac{1}{2}M^2X^2 + g^2X^2\Phi^\dagger\Phi - \frac{\lambda}{4}X^4$$

Options:

- freeze-out: sufficiently large g^2

$$v\sigma_{hh\to XX}\times n_h \gtrsim H \;\;\to\;\; \Omega_X \propto \frac{1}{\sigma_0}, \;\text{with}\; \frac{g^4}{(4\pi...)^2M^2} = \sigma_0 \equiv \sigma v$$

- freeze-in: intermediate g^2

$$\dot{n}_X + 3Hn_X = \sigma_{hh\to XX}n_h^2 \;\;\to\;\; \frac{n_X}{s} = \#\int dT\,\frac{n_h^2}{sHT}\times\frac{g^4}{T^2} \sim g^4\frac{M_{Pl}}{M} \;\;\to$$

$\Omega_X \propto g^4 \;\;\to\;\; g^2 \approx 10^{-11}$ still natural...

Freeze in via gravitational scatterings..?

any particles A in plasma

$$\sigma_{AA\to XX} \propto \frac{T^2}{M_{Pl}^4} \;\to\; \Omega_X \propto M_X \frac{T_i^3}{M_{Pl}^3}\dots$$

assuming $m \ll T_i$

called “unnatural” being dependent on the initial conditions