Flavour in Standard Model

Alex Bondar

- Short introduction in Standard Model
- Cabibbo-Kobayashi-Maskawa matrix, unitarity triangles
- CP violation
- Mixing, CPV in mixing
- Direct CPV
- Constraints on the unitarity triangle
- CPV in interference of mixing and decays, angle β
- B factories
- Mixing and CPV in Bs
- Angle γ
- LHCb detector
- Rear decays of B mesons, search of New Physics
- Future prospects

A Few Introductory Remarks

In my lectures I would like to focus not only on describing the current status of our understanding of flavor physics, but also on recalling the history of discoveries, brilliant ideas and misconceptions that eventually led to what we today call the Standard Model.

What I find particularly interesting is the close connection between experiment and theoretical ideas that ultimately led to modern understanding.

When preparing my lectures, I used a large number of previously given lectures on this topic by colleagues at various schools and conferences.

M. Blanke, Introduction to Flavour Physics and CP Violation, in Proceedings, 2016 European School of High-Energy Physics (ESHEP2016): Skeikampen, Norway, June 15-28 2016, pp. 71–100, 2017, 1704.03753.

Y. Nir, Probing new physics with flavor physics (and probing flavor physics with new physics), in Prospects in Theoretical Physics (PiTP) summer program on The Standard Model and Beyond IAS, Princeton, NJ, June 16-27, 2007, 2007, 0708.1872.

J. Zupan, Introduction to Flavour Physics, March 12, 2019, Published in: CERN Yellow Rep.School Proc. 6 (2019) 181-212,1903.05062

M. Vysotsky, Flavour Physics and CP violation, Dec 17, 2019, Published in: CERN Yellow Rep.School Proc. 5 (2022) 47, 1912.08717

S.Olsen, The Curious Early History of CKM Matrix, 2309.06042

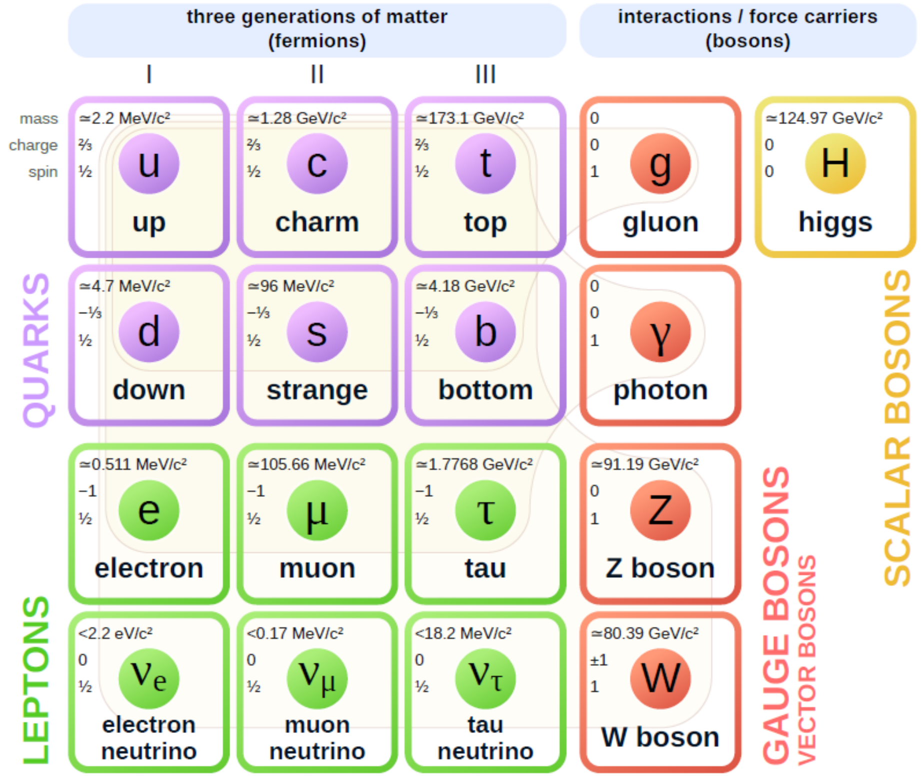
T.D.Lee, The Weak Interaction: It’s History and Impact on Physics, International Journal of Modern Physics A, Vol. 16, 22 (2001) 3633

Three generations of quarks (and leptons)

- identical gauge quantum numbers
- different masses

➢ **flavour physics** describes interactions that distinguish between flavours

$$\mathcal{G}_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$

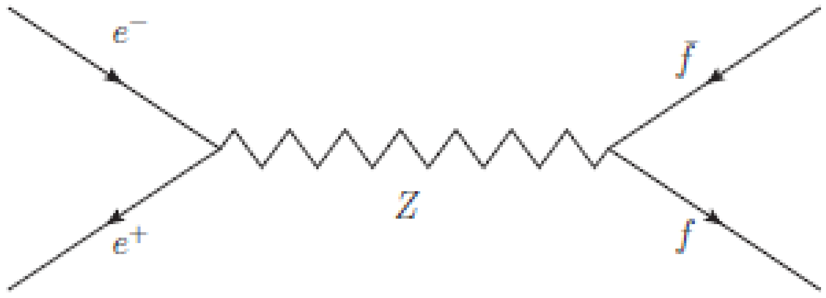


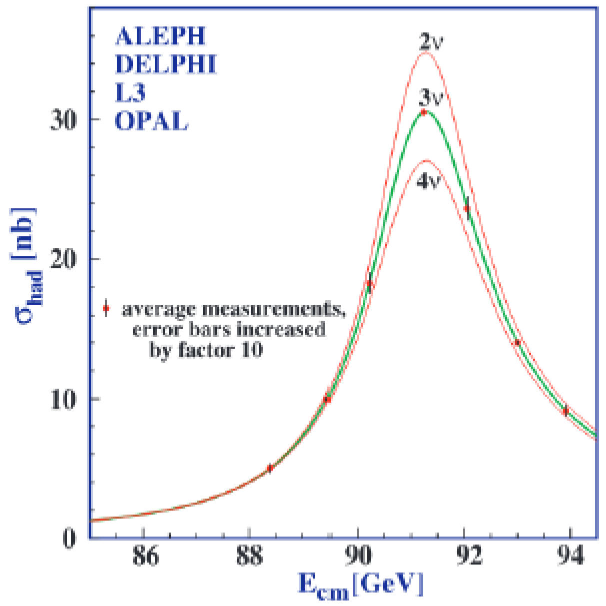
$SU(3)_c$ is the gauge group of strong interactions, (QCD),
$SU(2)_L$ is the gauge group of weak isospin,
$U(1)_Y$ the gauge group of hypercharge.

The $\mathcal{G}_{SM}$ is spontaneously broken by the Higgs vacuum expectation value, $\langle H \rangle = \left(0, \frac{\upsilon}{\sqrt{2}}\right), \upsilon = 246\ GeV$, down to

$$\mathcal{G}_{SM} \to SU(3) \times U(1)_{em}$$

Why only 3 lepton generations?





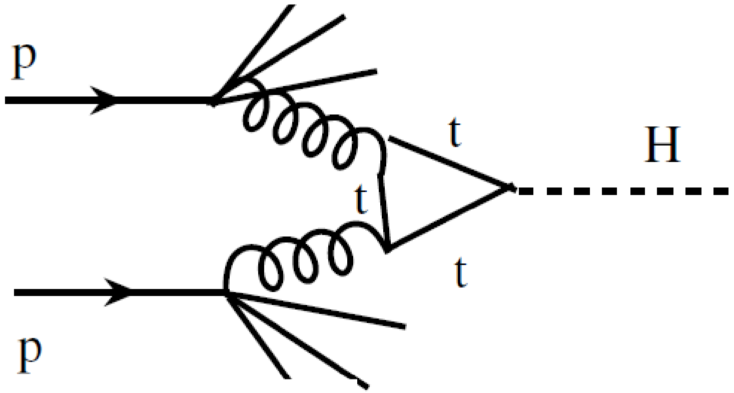
$$\Gamma_{Z\to f\bar{f}} = \frac{G_F M_Z^3}{6\sqrt{2}\pi}[(g_V^f)^2 + (g_A^f)^2] = 332[(g_V^f)^2 + (g_A^f)^2] \text{ MeV}$$

$$\Gamma_{Z\to\nu\nu}^{\text{theor}} = 3\cdot 332[\frac{1}{4}+\frac{1}{4}] = 498 \text{ MeV}$$

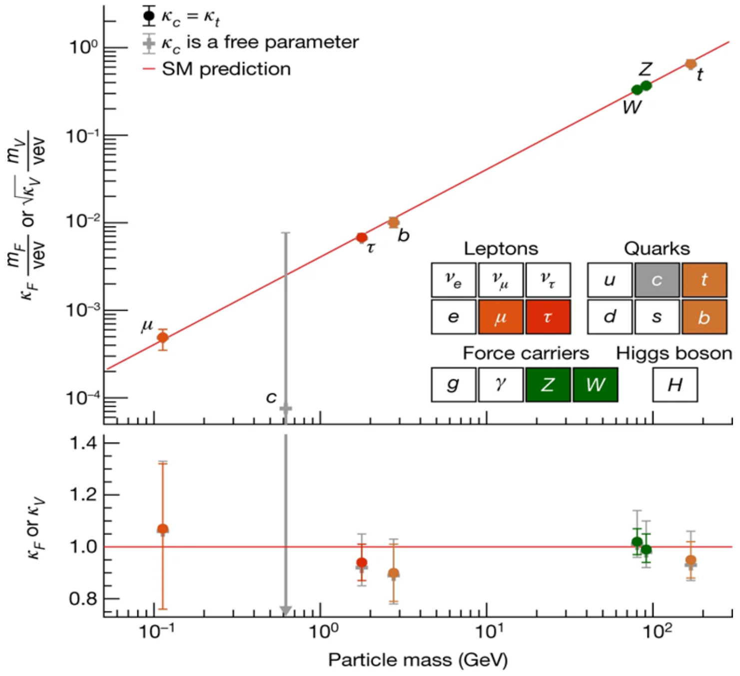
$$\Gamma_{inv}^{\text{exp}} = 499 \pm 1.5 \text{ MeV}$$

Why only 3 quark generations?

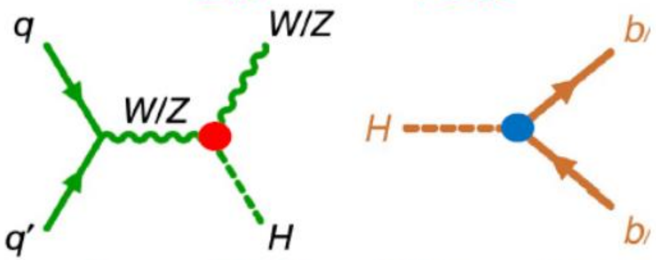
In H production at LHC the following diagram dominates:



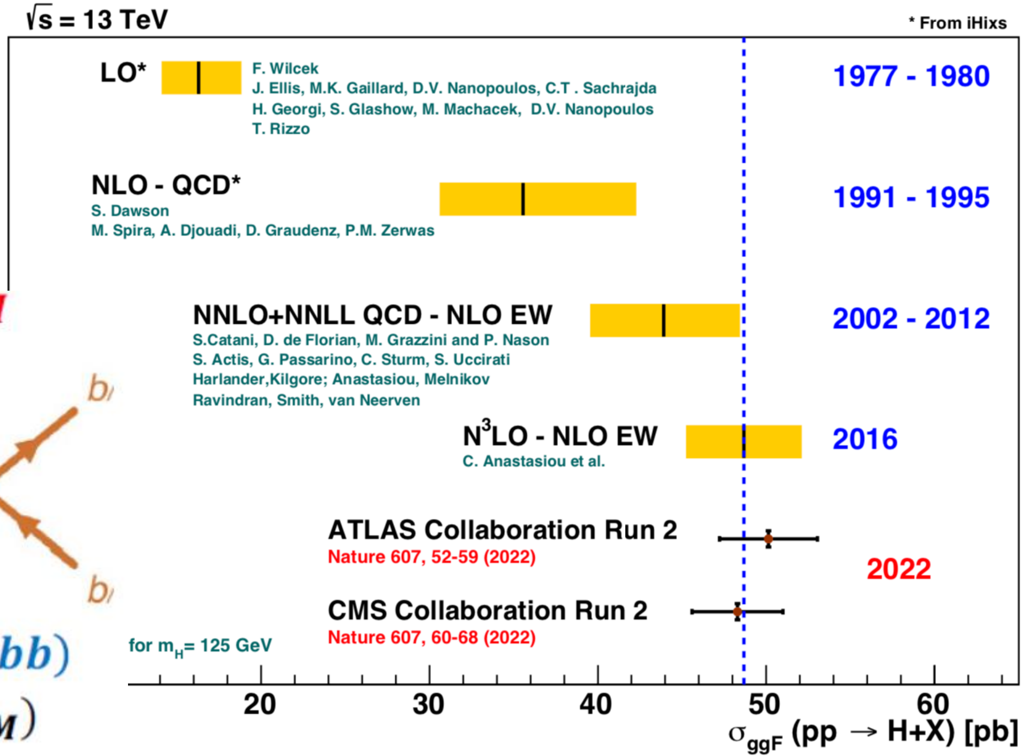
and for $2m_t >> M_H$ the corresponding amplitude does not depend on m_t.
In case of the 4th generation $T-$ and $B-$ quarks contribute, so the amplitude triples and the cross section of H production at LHC becomes 9 times larger than in SM, which is definitely excluded.



$$\kappa = g_x^{measure} / g_x^{SM}$$



$$\sigma(pp \to VH) \cdot BR(H \to bb) = k_{W/Z}^2 \cdot k_b^2 \times (\sigma_{SM} \cdot BR_{SM})$$

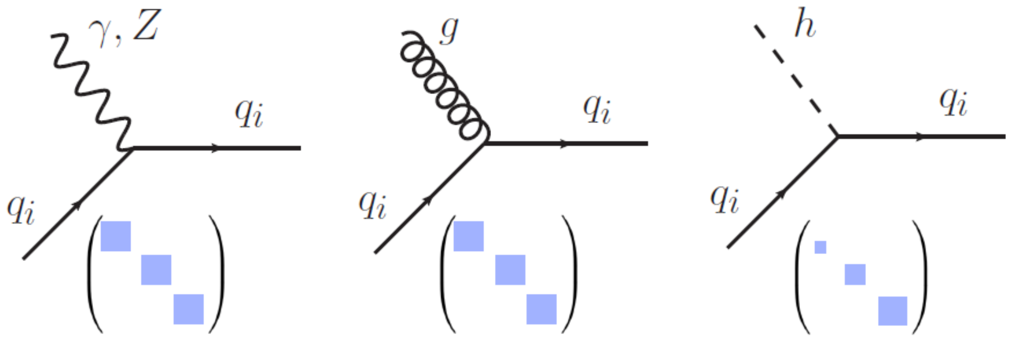


SM Lagrangian for the quarks in the mass basis

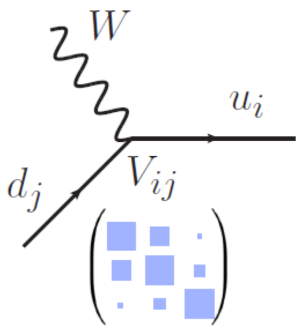
$$\mathcal{L}_{SM} \sim (\bar{q}_i D_{NC} q_i) + \frac{g}{\sqrt{2}} \bar{u}_{Li} \gamma_\mu W^{\mu+} V_{CKM}^{ij} d_{Lj} + m_{u_i} \bar{u}_L^i u_R^i + m_{d_i} \bar{d}_L^i d_R^i + h.c.$$

The covariant derivative D_{NC} contains flavour universal couplings of photon, gluon and the Z

The Higgs has flavour diagonal, yet non-universal, couplings that are proportional to quark masses



The flavour changing transitions reside in charged currents with the strength encoded in the CKM matrix



The CKM matrix

transformations of the right-handed quarks are indeed unphysical, i. e. they leave the rest of the Lagrangian invariant

however, u_{Li} and d_{Li} form the $SU(2)_L$ doublets Q_{Li}

➢ kinetic term gives rise to the interaction

$$\frac{g}{\sqrt{2}}\bar{u}_{Li}\gamma_\mu W^{\mu+} d_{Li}$$

transforming to the mass eigenstate basis, we obtain

$$\frac{g}{\sqrt{2}}\bar{u}_{Li}\hat{U}^\dagger_{L,ij}\hat{D}_{L,jk}\gamma_\mu W^{\mu+} d_{Lk}$$

$$Q_{Li} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

In the SM, the left-handed quarks are arranged in doublets of the $SU(2)_L$ weak interactions while the right-handed quarks are introduced as $SU(2)_L$ singlets:

$$U_j = u_R, c_R, t_R$$

$$D_j = d_R, s_R, b_R$$

The combination $\hat{V}_{\text{CKM}} = \hat{U}^\dagger_L \hat{D}_L$ is physical and is called the CKM matrix. It leads to flavour violating charged current interactions.

Parameter counting

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

How many parameters does the CKM matrix have?

unitary 3×3 matrix can be parametrised by 3 mixing angles and 6 complex phases.

however 5 phases are unphysical, as they can be absorbed as unobservable parameters into the up-type and down-type quarks, respectively

note: overall phase rotation of *all* quarks does not affect the CKM matrix

CKM matrix contains three mixing angles and one physical complex phase

Standard parametrisation $(s_{ij} = \sin\theta_{ij}, c_{ij} = \cos\theta_{ij})$

$$V_{\text{CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

How did we come to this knowledge?

β decay

In 1898 Lord Rutherford discovered that the so-called Becquerel ray actually consisted of two distinct components: one that is readily absorbed, which he called alpha radiation, and another of a more penetrating character, which he called beta radiation.

Philos. Mag. 42, 392 (1898)

In 1900 Curies measured the electric charge of the particle and found it to be negative.

C. R. Acad. Sci. 130, 647 (1900)

In 1908, Hahn and Meitner published a paper stating that the β ray carries a unique energy. Their evidence was that the absorption curve of a β ray shows an exponential decrease along its path when passing through matter, like the α ray.

Phys. Z. 9, 321, 697 (1908)

Wilson in 1909 found electrons to exhibit totally different behavior from the α particle

Proc. Roy. Soc. A82, 612 (1909)

Only in 1922 it was demonstrated by Ellis that the β energy indeed continuous. Furthermore, Ellis proved that the β maximum energy equals the difference of the initial and final nuclear energy.

Proc. Cambridge Philos. Soc. 21, 121 (1922)

Neutrino

Bohr proposed the hypothesis of nonconservation of energy in nuclear decay

J. Chem. Soc. 135, 349 (1932)

Pauli suggested that in the decay energy is conserved, but accompanying the particle there is always emission of a neutral particle of extremely small mass and with almost no interaction with matter. Since such a weakly interacting neutral particle is not detected, there appears to be an apparent nonconservation of energy.

Letter of December 4, 1930

"…the possibility that there could exist in the nuclei electrically neutral particles, that I wish to call "neutrons", which have spin 1/2 and obey the exclusion principle and which further differ from light quanta in that they do not travel with the velocity of light. The mass of the "neutrons" should be of the same order of magnitude as the electron mass and in any event not larger than 0.01 proton masses. The continuous beta spectrum would then become understandable by the assumption that in beta decay a "neutron" is emitted in addition to the electron such that the sum of the energies of the "neutron" and the electron is constant..."

Fermi theory of the β decay

In 1934 Fermi motivated by Electrodynamic proposed the Theory of Weak Interactions. This in turn stimulated further investigation of the spectrum shape of the decay, which did not agree with Fermi's theoretical prediction.

Nuovo Cimento 11, 1 (1934); Z. Phys. 88, 161 (1934).

Fermi's original vector-coupling form,

$$G(\psi_n^\dagger \gamma_4 \gamma_\lambda \psi_p)(\psi_e^\dagger \gamma_4 \gamma_\lambda \gamma_5 \psi_\nu)$$

was too simple; to conform to reality, it should be extended to include a Gamow-Teller term. It is curious why Fermi should choose this particular expression, which resembles the V-A interaction, but with parity conservation.

Experimental situation with β decays was finally clarify by Albert and Wu in 1949

Phys. Rev. 75, 315 (1949)

Universal Fermi Interaction

1948, Klein's idea that μ decay and β decay can be described by the same four-fermion interaction

Nature 161, 897 (1948)

> Since, according to the above assumptions, the decay of the ordinary meson is, so to speak, the prototype of all β-processes, it is important that the value of the life-time, $\tau = 2 \times 10^{-6}$ sec., and the energy available in the process $\sim 100\ m_e c^2$, fit in very well with the value to be expected from our knowledge of the β-decay.

T. D. Lee, M. Rosenbluth and C. N. Yang suggested in analogy with electromagnetic forces, the basic weak interaction could be carried by a universal coupling through an intermediate heavy boson

Phys. Rev. 75, 905 (1949)

$\theta - \tau$ Problem

In the early 1950s, θ referred to the charged meson which decays into 2π, whereas τ referred to the one decaying into 3π . The spin-parity of θ is clearly $0^+, 1^-, 2^+$,etc. By 1954 existing data (Dalitz plot) were more consistent with the assignment 0^-than 1^-.

Both mesons were known to have comparable masses, but mass was very close to three times the pion mass. So the phase space available for θ decay was much bigger than that for τ decay, therefore one can expect the θ should have a much less lifetime. However, when accurate lifetime measurements were made in 1955, it turned out that θ and τ have the same lifetime.

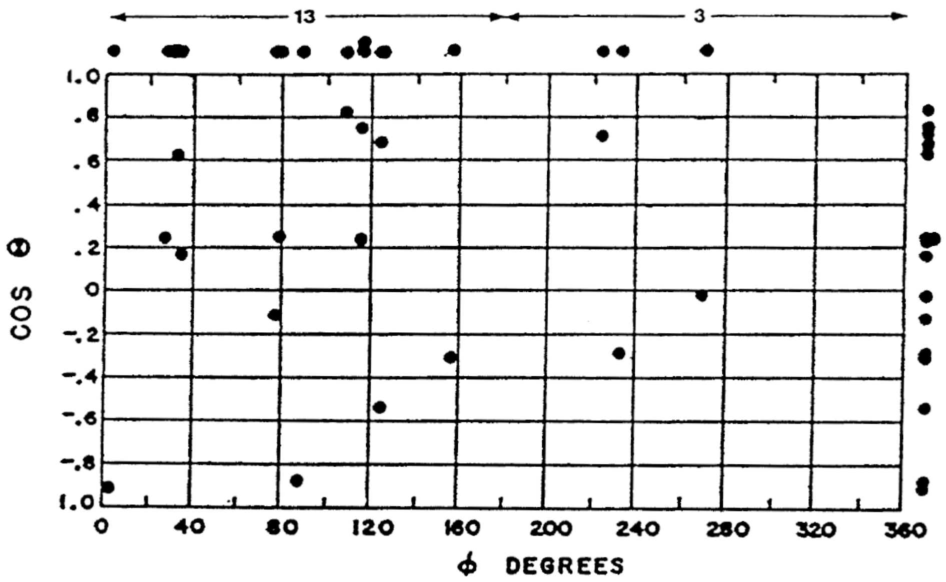
This presented a very puzzling picture

$\theta - \tau$ Problem

At the Rochester Conference on high energy physics (April, 1956) Steinberger reported the study of the strange particles pairs production in the reaction $\pi^- p \rightarrow \Sigma^- \theta^+ \rightarrow n\pi^- \theta^+$ in order to defined the spin Σ^-. They studied $\cos(\phi) \sim (\vec{p}_\pi \times \vec{p}_\Sigma)(\vec{p}_\Sigma \times \vec{p}_n)$ distribution.

Just after Conference T.D.Lee suggested for Steinberger to use another combination $\sin(\phi) \sim (\vec{p}_{\perp\pi} \times \vec{p}_{\perp n})\vec{p}_\Sigma$, which is P-odd.

Int. J. of Modern Physics A, Vol. 16, No. 22 (2001) 3633



Phys. Rev. 103, 1827 (1956)

Lederman et al Observation of the two neutral strange particles θ_1^0 and θ_2^0

Phys. Rev. 103, 1901 (1956)

Parity Nonconservation in Weak Decays

In 1956 Lee and Yang came to conclusion that the weak interactions did not conserved parity – largely on the basis of the fact that the K^+ could decay in two decay modes $K^+ \to 2\pi$ and $K^+ \to 3\pi$, in which the final states have opposite parities.

PR 104, 256 1956

This was unacceptable for many people, including Landau: empty space has left-right interchange symmetry, so a Lagrangian should have it as well.

Ioffe, Okun and Rudik noted that Lee and Yang's theory violates charge conjugation symmetry (C) as well, while CP is conserved explaining the difference of life times of K_L and K_S

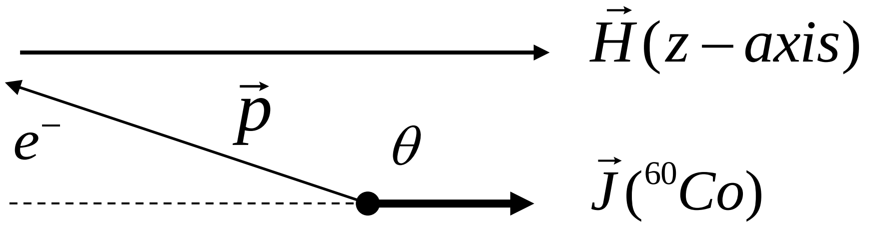
JETP 5,328 1957

Landau found the way to resurrect P-invariance stating that the theory should be invariant under the product of P reflection and C conjugation. He called this product the combined inversion and according to him it should substitute P-inversion broken in weak interactions. In this way the theory should be invariant when together with changing the sign of the coordinate, one changes all particles to antiparticles. Combined parity instead of parity.

JETP 5,336 1957

Parity Nonconservation in weak decays

The test parity conservation was performed by Wu et. al (1957).



PR 105,1413
1957

^{60}Co(J=5) decays to ^{60}Ni*(J=4). The relative electron intensities along and against the field direction were measured

As a result $$I(\theta) = 1 + \alpha \frac{\vec{\sigma} \cdot \vec{p}}{E} = 1 + \alpha \frac{v}{c} \cos\theta$$

Where $\alpha = -1$

Feynman and Gell-Mann proposed that the weak interaction was a current-current V-A interaction and the currents for the ΔS = 0 and ΔS = ±1 hadronic transitions and the lepton currents all have a common coupling strength.

PR 109,193
1958

Cabibbo pointed out that Feynman-Gell-Mann universality conjecture failed miserably and suggested two coupling constants α,β for ΔS = 0 and ΔS = ±1 hadronic currents such as $\alpha^2+\beta^2=1$

PRL 10,531
1963

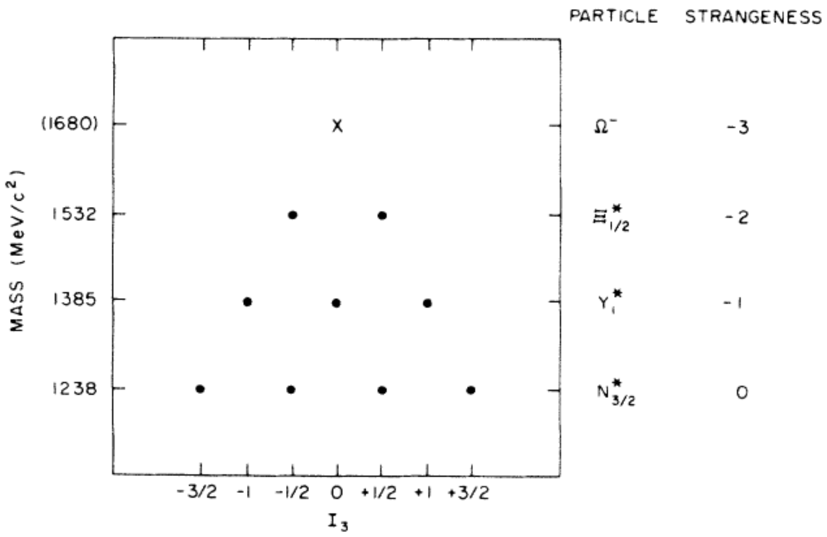
Quarks

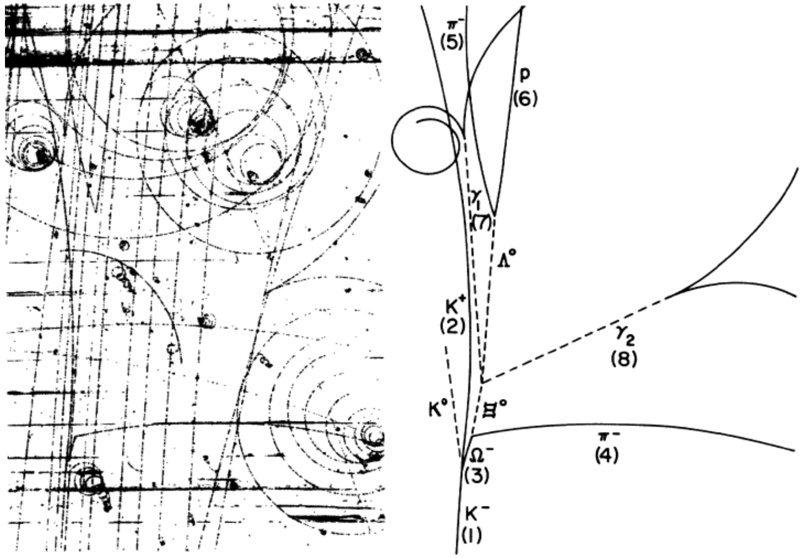
PL 8, 214
1964
CERN-TH-401
unpublished

Gell-Mann and Zweig proposed the quark model in which hadrons were comprised of fractionally charge fermionic constituents

Barnes et al Observation of a hyperon with strangeness three at BNL 80-inch hydrogen bubble chamber

PRL 12, 204
1964





$K^- + p \to \Omega^- + K^+ + K^0$

$\quad \hookrightarrow \Xi^0 + \pi^-$

$\quad\quad \hookrightarrow \Lambda^0 + \pi^0$

$\quad\quad\quad \hookrightarrow \gamma_1 + \gamma_2$

$\quad\quad\quad\quad \hookrightarrow e^+ + e^-$

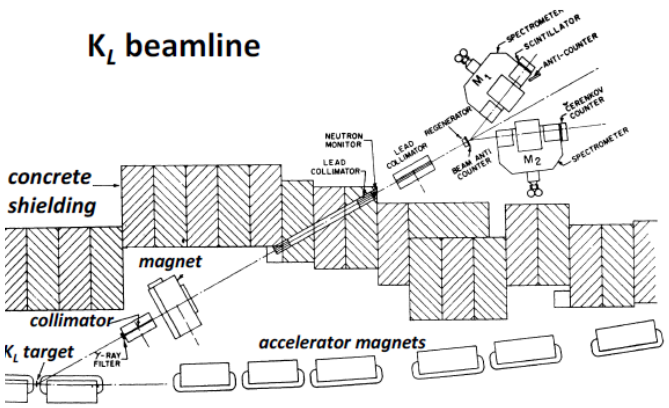
$\quad\quad\quad\quad \hookrightarrow e^+ + e^-$

$\quad\quad \hookrightarrow \pi^- + p.$

CPV discovery

PRL 13, 138
1964

Christenson, Cronin, Fitch and Turlay observed 45 events of the $K^0_L \to \pi^+\pi^-$ decay $(2\pm0.4)10^{-3}$



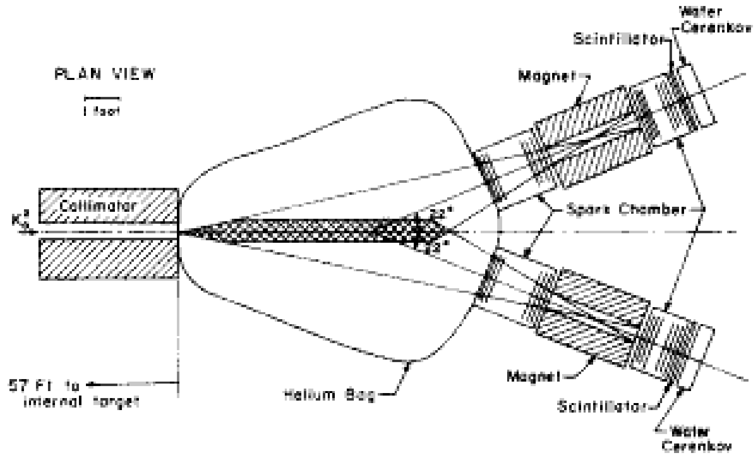
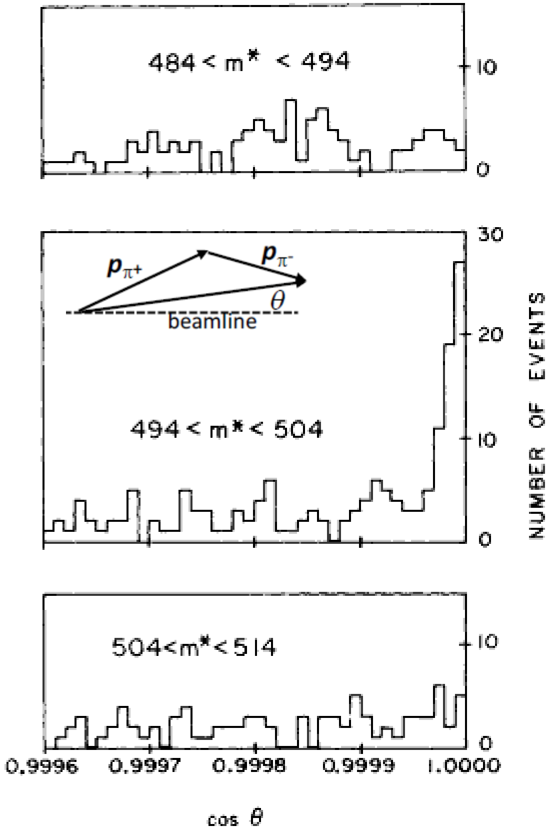


FIG. 1. Plan view of the detector arrangement.



This result was reported by Fitch at ICHEP1964 (Dubna) in August. At the same conference Okonov presented upper limit on the $K^0_L \to \pi^+\pi^-$ decay $<2.5\ 10^{-3}$ from Dubna experiment.

Why it's so important

Sakharov realized that CP violation is one of the necessary conditions of the excess of matter over antimatter in the Universe

JETP Lett. 6, 21 1967

The baryon asymmetry of the Universe is the measurement of

$$\frac{n_B - n_{\bar{B}}}{n_\gamma} \sim 10^{-10}$$

This means that 10^{-6} seconds after the Big Bang, when the temperature was T > 1 GeV, and quarks and antiquarks were in thermal equilibrium, there was a corresponding asymmetry between quarks and antiquarks.

Sakharov pointed out that for a theory to generate such an asymmetry in the course of its evolution from a hot Big Bang (assuming inflation washed out any possible prior asymmetry), it must contain:

(1) baryon number violating interactions;

(2) C and CP violation;

(3) deviation from thermal equilibrium.

Interestingly, the SM contains all three conditions, but CP violation is too small, and the deviation from thermal equilibrium is too small at the electroweak phase transition.

GIM mechanism

Glashou, Iliopoulos, Maiani Weak Interactions with Lepton-Hadron Symmetry

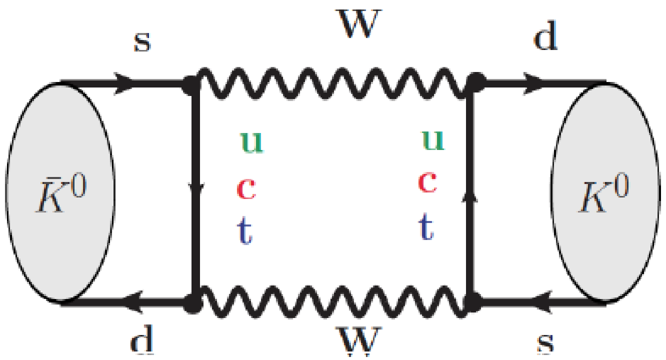
PRD v2, n7 1285 1970

Flavor change neutral current suppression due to unitarity in scheme with four quarks

$$\begin{pmatrix} u \\ d \end{pmatrix} s \Longrightarrow \begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix}$$

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

unitarity: $\sum_{i=u,c,t} V_{id}^* V_{is} = 0$



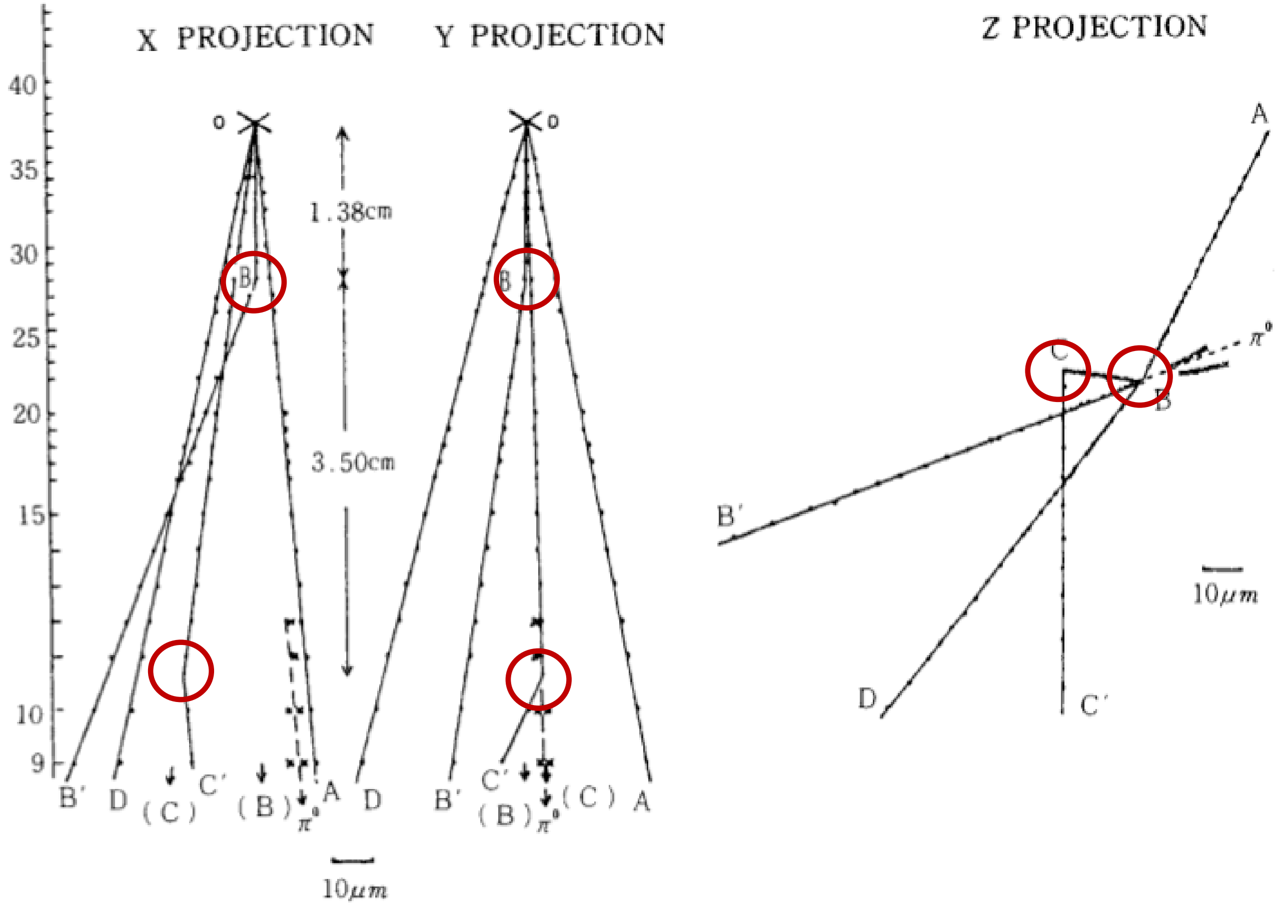
$$\mathcal{M} \propto \sum_{i,j=u,c,t} V_{id}^* V_{is} V_{jd}^* V_{js} F(x_i, x_j)$$

$F(x_i, x_j)$: loop function that depends on mass square ratios $x_i = m_i^2/M_W^2$

Kiyoshi Niu event

In 1970, a small team of experimenters in Japan led by Kiyoshi Niu, exposed a stack of photographic emulsions to cosmic rays in a high altitude commercial cargo airliner. They found a remarkable event, in which an ultra-high energy cosmic rays particle produced long lived particles with large masses.

Prog.Theor.Phys. 46, 1644, 1971



Assumed decay mode	Mass (GeV)	lifetime (sec)
$B \to \pi^+ \pi^0$	1.78	2.2×10^{-14}
$C \to (\pi^0) p$	2.95	3.6×10^{-14},

Kobayashi Maskawa quark mixing

Kobayashi, Maskawa CP-Violation in the Renormalizable Theory of Weak Interaction

Prog.Theor.Phys. 49, 652, 1973

Eureka! With six-quarks there is room for a CP-violating phase!

Next we consider a 6-plet model, another interesting model of CP-violation. Suppose that 6-plet with charges $(Q, Q, Q, Q-1, Q-1, Q-1)$ is decomposed into $SU_{\text{weak}}(2)$ multiplets as $2+2+2$ and $1+1+1+1+1+1$ for left and right components, respectively. Just as the case of (A, C), we have a similar expression for the charged weak current with a 3×3 instead of 2×2 unitary matrix in Eq. (5). As was pointed out, in this case we cannot absorb all phases of matrix elements into the phase convention and can take, for example, the following expression:

$$\begin{pmatrix} \cos\theta_1 & -\sin\theta_1\cos\theta_3 & -\sin\theta_1\sin\theta_3 \\ \sin\theta_1\cos\theta_2 & \cos\theta_1\cos\theta_2\cos\theta_3-\sin\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\cos\theta_2\sin\theta_3+\sin\theta_2\cos\theta_3 e^{i\delta} \\ \sin\theta_1\sin\theta_2 & \cos\theta_1\sin\theta_2\cos\theta_3+\cos\theta_2\sin\theta_3 e^{i\delta} & \cos\theta_1\sin\theta_2\sin\theta_3-\cos\theta_2\sin\theta_3 e^{i\delta} \end{pmatrix}. \quad (13)$$

$$\theta_1, \theta_2, \theta_3, \delta \longrightarrow 0 \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Then, we have CP-violating effects through the interference among these different current components. An interesting feature of this model is that the CP-violating effects of lowest order appear only in $\Delta S\neq 0$ non-leptonic processes and in the semi-leptonic decay of neutral strange mesons (we are not concerned with higher states with the new quantum number) and not in the other semi-leptonic, $\Delta S=0$ non-leptonic and pure-leptonic processes.

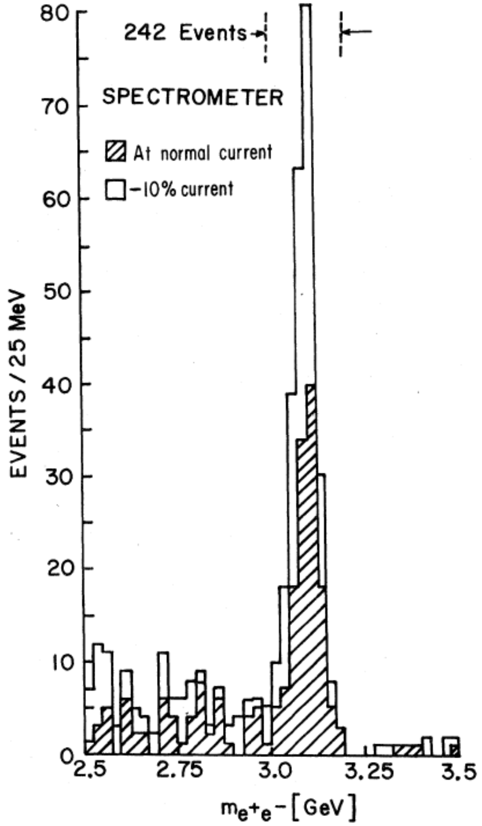
November 1974 Revolution

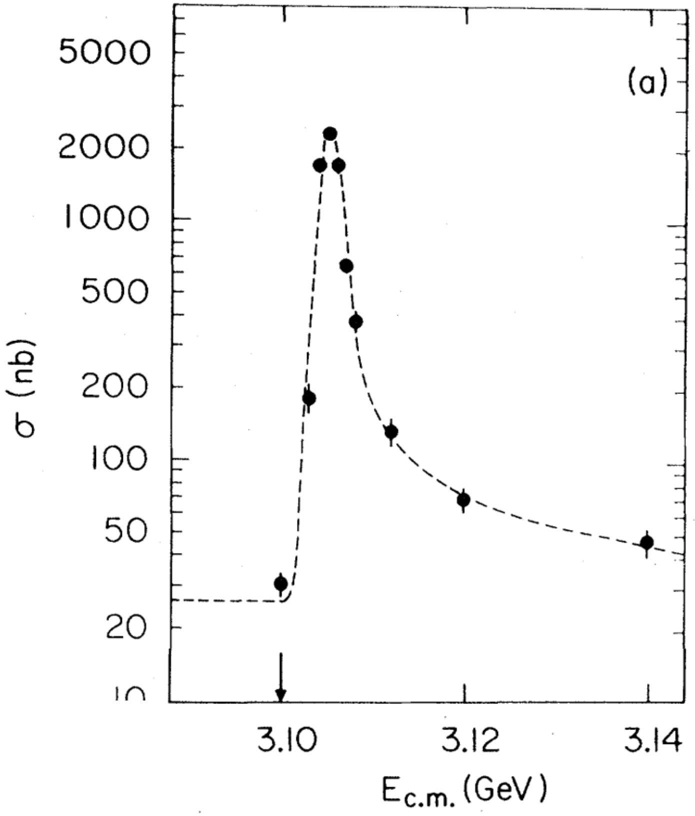
Aubert, … Ting, et al Experimental Observation of a Heavy Particle J

PRL 33, 1404 1974

Augustin, … Richter, et al Discovery of a Narrow Resonance in e^+e^- Annihilation

PRL 33, 1406 1974





Discovery of the third generation

Perl, et al Evidence for Anomalous Lepton Production in e+e- Annihilation

PRL 35, 1489
1975

Discovery of the $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \mu^+\nu_\mu\bar{\nu}_\tau e^-\bar{\nu}_e\nu_\tau$

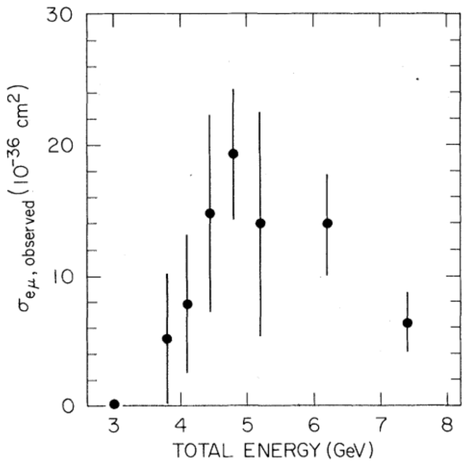
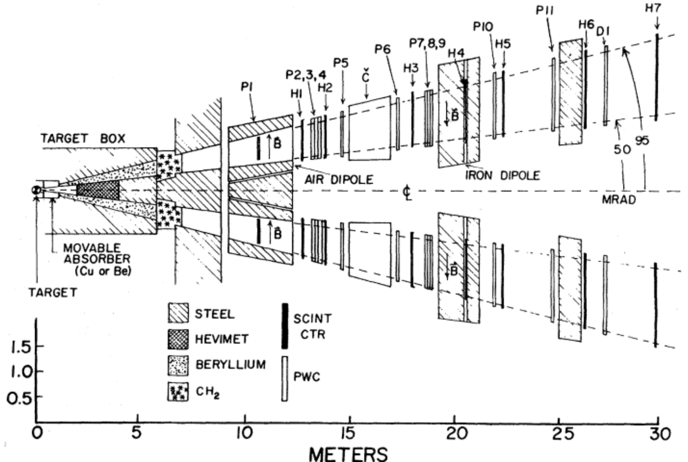


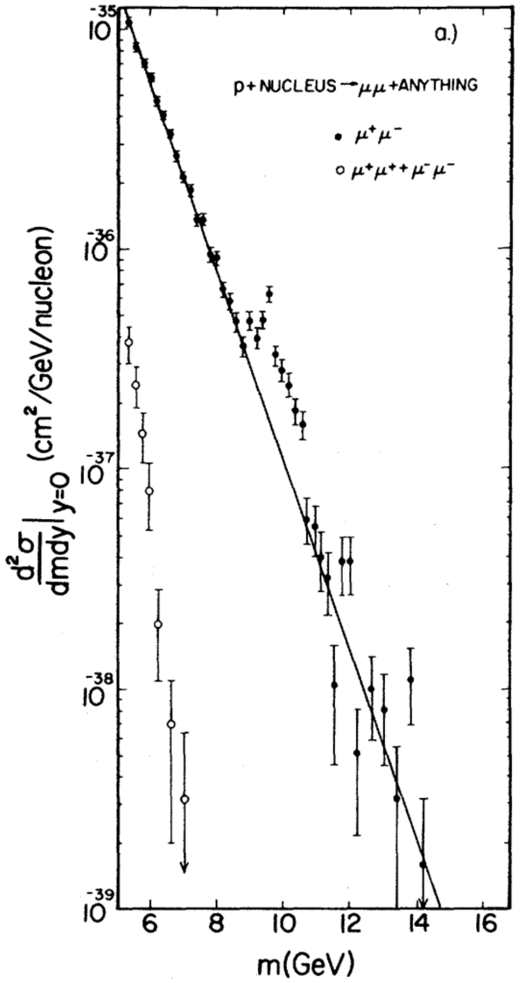
FIG. 2. The *observed* cross section for the signature e-μ events.

Herb,…Lederman et al Observation of a Dimuon Resonance at 9.5 GeV in 400-GeV Proton-Nucleus

PRL 39, 252
1977



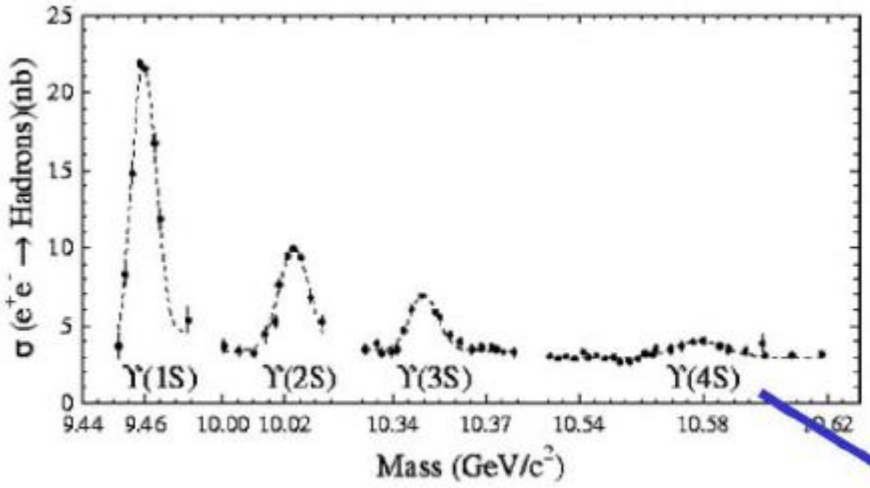
Discovery of the $\Upsilon(9.46) \rightarrow \mu^+\mu^-$ interpreted as $^3S_1\ b\bar{b}$



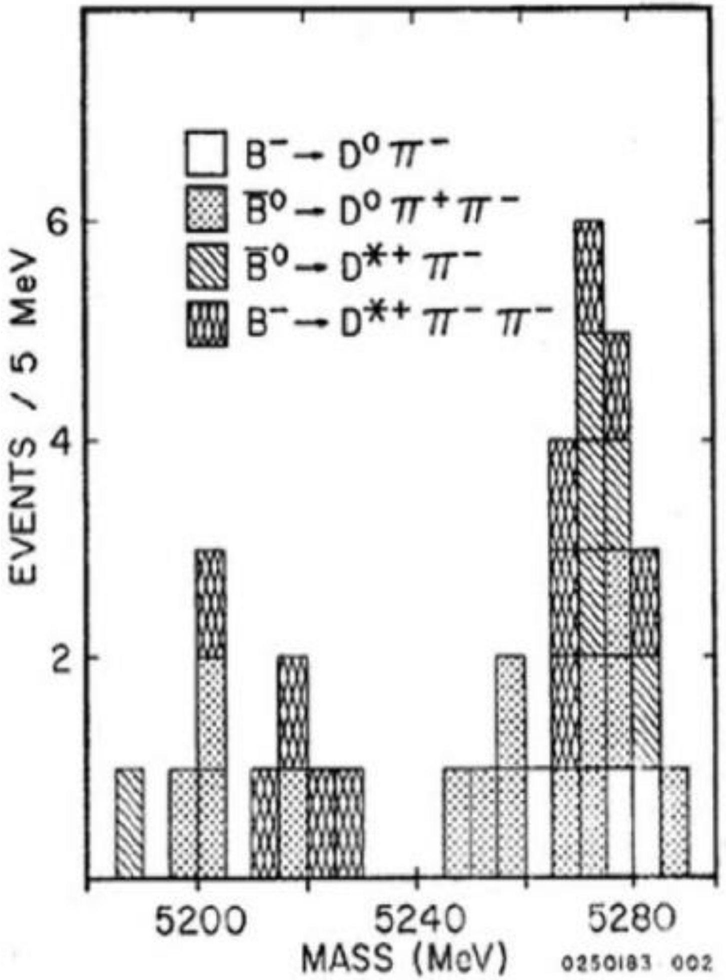
B mesons production at e^+e^- colliders

CLEO Collaboration Observation of Exclusive Decay Modes of b-Flavored Mesons

PRL 50, 881
1983



CESR at Cornell:
"naked beauty"



b lifetime

MAC Collaboration

MARK II Collaboration

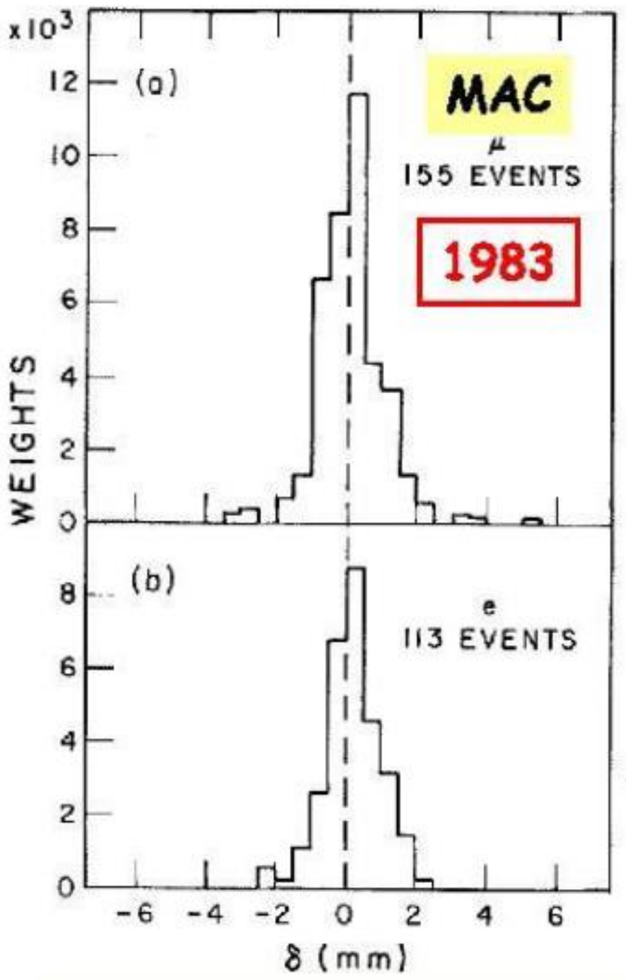
PRL 51, 1022 1983
PRL 51, 1316 1983

Isolate samples of high-p_T leptons (155 muons, 113 electrons) wrt thrust axis

- Measure impact parameter δ wrt interaction point
- Signed by taking thrust axis of *b*-jet as the *B* hadron direction

Lifetime implies V_{cb} small

- MAC: $(1.8 \pm 0.6 \pm 0.4)$ ps
- Mark II: $(1.2 \pm 0.4 \pm 0.3)$ ps



CKM matrix Wolfenstein parameterization

$$\begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+\frac{1}{2}A^2\lambda^5[1-2(\rho+i\eta)] & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3[1-(1-\frac{1}{2}\lambda^2)(\rho+i\eta)] & -A\lambda^2+\frac{1}{2}A\lambda^4[1-2(\rho+i\eta)] & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} \begin{matrix} u \\ c \\ t \end{matrix}$$

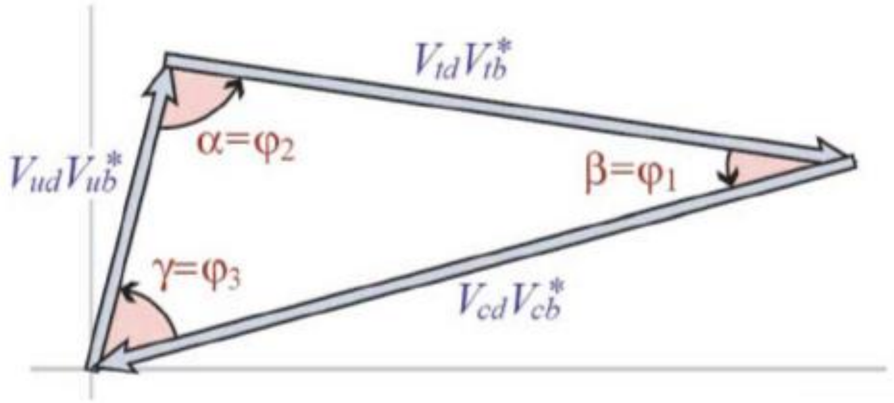
$$\begin{matrix} d & s & b \end{matrix}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0\,.$$

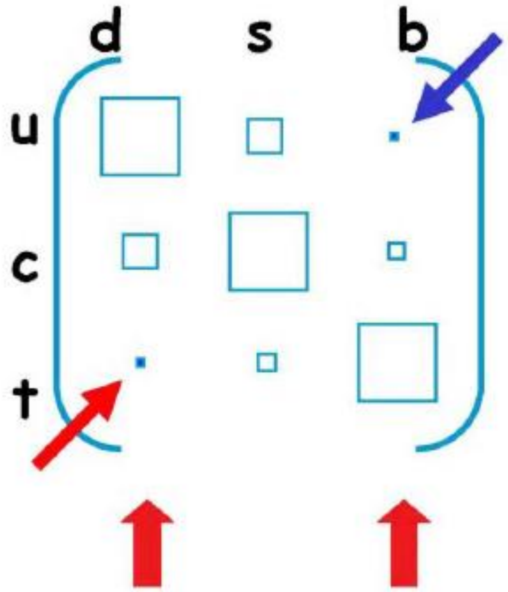
$$\alpha \equiv \varphi_2 \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right),$$

$$\beta \equiv \varphi_1 \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right),$$

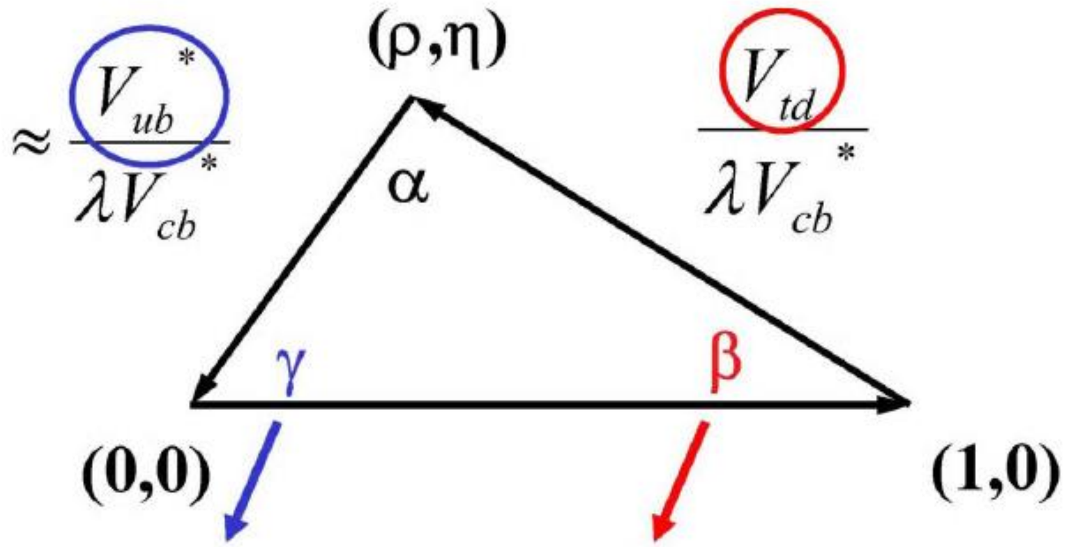
$$\gamma \equiv \varphi_3 \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right),$$



Unitarity Triangle



apply unitarity constraint to these two columns



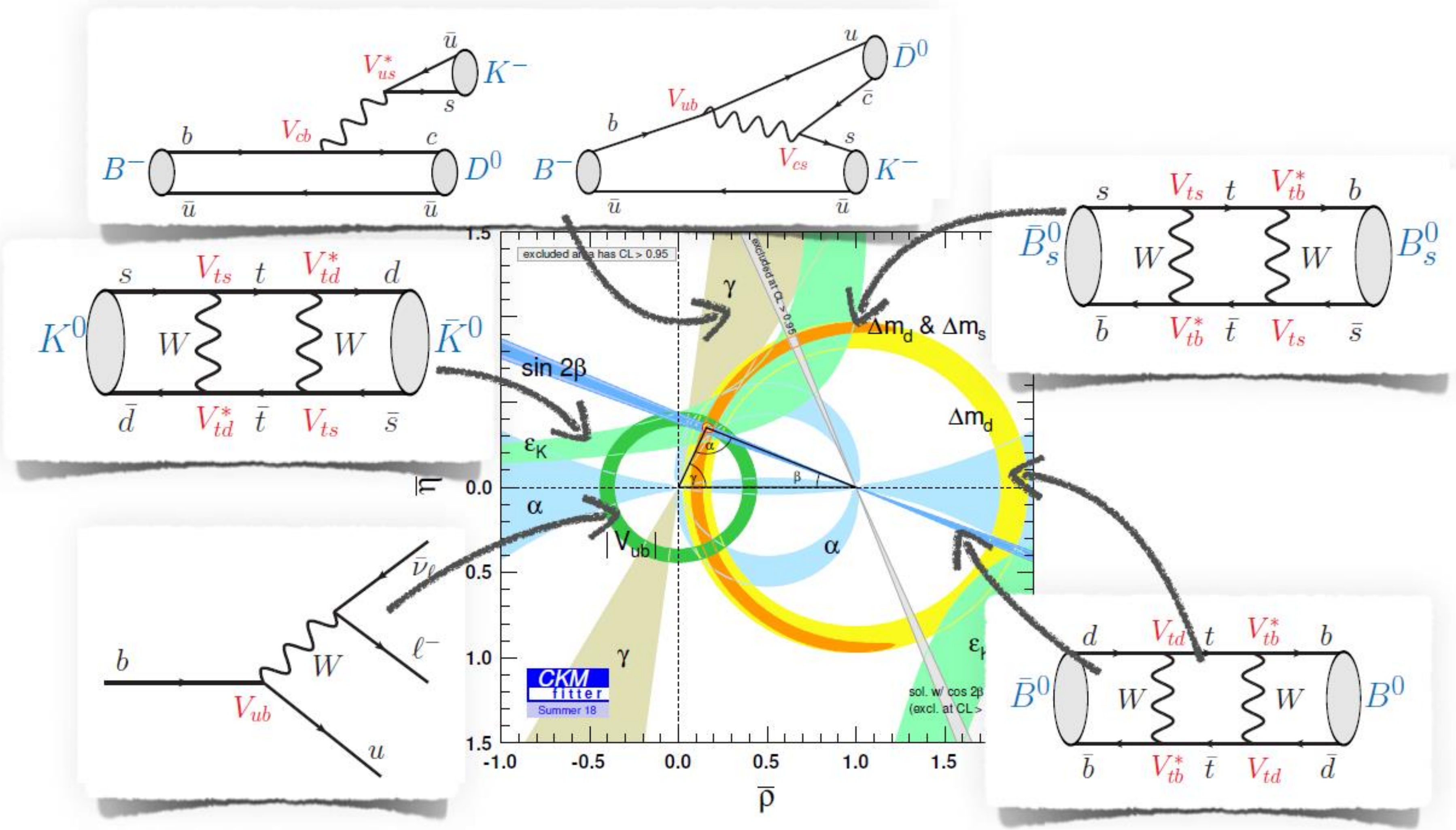
$$\gamma \approx \arg V_{ub}^* \quad \beta \approx -\arg V_{td} \quad \alpha = \pi - \beta - \gamma$$

Orders of magnitude for Wolfenstein parameters:

$$\lambda \approx 0.22\,, \quad A \approx 0.8\,, \quad \sqrt{\rho^2 + \eta^2} \approx 0.4$$

$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$

$$V_{cd} = \lambda\,, \quad V_{ud} \approx V_{tb} \approx 1$$


excluded area has CL > 0.95
excluded at CL > 0.95
γ
Δm_d & Δm_s
sin 2β
Δm_d
ε_K
α
β
|V_ub|
α
γ
sol. w/ cos 2β
(excl. at CL >
CKM fitter
Summer 18
-1.0 -0.5 0.0 0.5 1.0 1.5 2.0
ρ̄
η̄
B⁻ D⁰ K⁻ D̄⁰ K⁰ K̄⁰ B̄_s⁰ B_s⁰ B̄⁰ B⁰
V_cb V*_us V_ub V_cs V_ts V*_td V*_tb V_td
W

Quarks in the SM

Parity violation of electroweak interactions

In the SM, the left-handed quarks are arranged in doublets of the $SU(2)_L$ weak interactions:

$$Q_j = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

while the right-handed quarks are introduced as $SU(2)_L$ singlets:

$$U_j = u_R, c_R, t_R$$

$$D_j = d_R, s_R, b_R$$

The quarks' couplings to the gluons, weak gauge bosons $W^{\pm}$ and Z, and the photon are described by the kinetic term in the Lagrangian

$$\mathcal{L}_{\text{fermion}} = \sum_{j=1}^{3} \bar{Q}_j i \not{D}_Q Q_j + \bar{U}_j i \not{D}_U U_j + \bar{D}_J i \not{D}_D D_j$$

with the covariant derivatives

$$\begin{aligned} D_{Q,\mu} &= \partial_\mu + i g_s T^a G^a_\mu + i g \tau^a W^a_\mu + i Y_Q g' B_\mu \\ D_{U,\mu} &= \partial_\mu + i g_s T^a G^a_\mu + i Y_U g' B_\mu \\ D_{D,\mu} &= \partial_\mu + i g_s T^a G^a_\mu + i Y_D g' B_\mu \end{aligned}$$

and the hypercharges assigned as Q^Y_Q= 1/6, Q^Y_U = 2/3, Q^Y_D = −1/3. T^a(a = 1, . . . , 8) and τ^a (a = 1, 2, 3) are the generators of $SU(3)_c$ and $SU(2)_L$, respectively, and the index j runs over the three generations of quark fields. It is evident that the gauge couplings are universal for all three generations.

Yukawa couplings

Flavour non-universality, on the other hand, is introduced by the quark Yukawa couplings to the Higgs field, responsible for the generation of non-zero quark masses:

$$\mathcal{L}_{\mathsf{Yuk}} = \sum_{i,j=1}^{3} (-Y_{U,ij}\bar{Q}_{Li}\tilde{H}U_{Rj} - Y_{D,ij}\bar{Q}_{Li}HD_{Rj} + h.c.)$$

where h.c. abbreviates the hermitian conjugate term.

The subscripts i, j are generation indices, and the dual field $\widetilde{H}$ is given as $\widetilde{H} = \epsilon H^{*} = (H^{0*}, -H^{-})^{T}$.

Replacing the Higgs field H by its vacuum expectation value $\langle H \rangle = (0, v)^{T}$, we obtain the quark mass terms

$$\sum_{i,j=1}^{3} (-m_{U,ij}\bar{u}_{Li}u_{Rj} - m_{D,ij}\bar{d}_{Li}d_{Rj} + h.c.)$$

with the quark mass matrices given by $m_{U,D} = vY_{U,D}$.

The quark mass matrices m_U and m_D are 3×3 complex matrices in flavour space.
They can be diagonalized by making appropriate bi-unitary field redefinitions:

$$u_L = \widehat{U}_L u_L^m, \; u_R = \widehat{U}_R u_R^m, \;\; d_L = \widehat{D}_L d_L^m, \;\; d_R = \widehat{D}_R d_R^m,$$

with the superscript ^{m} denoting quarks in their mass eigenstate basis.